Pre-Calculus 120B Exam Review

Formulas

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{(n-r)!r!}$$

$$t_n = t_1 + d(n-1) \qquad S_n = \frac{n}{2}(t_1 + t_n) \quad \text{or} \quad S_n = \frac{n}{2}(2t_1 + d(n-1))$$

$$t_n = t_1 \times r^{n-1} \qquad S_n = \frac{t_1(1-r^n)}{1-r} \quad \text{or} \quad S_n = \frac{rt_n - t_1}{r-1}$$

$$S_\infty = \frac{t_1}{1-r}$$

SHORT RESPONSE:

1. How many six-character alphanumeric codes can be made without any repeats?

2. How many odd three-digit numbers can be made from using 2, 5, 6, 7 or 9, if repeats are allowed?

3. Evaluate $\frac{14!}{11!} + 7$.

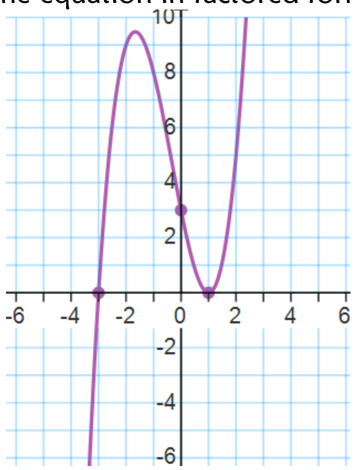
4. Express $34 \times 33 \times 32 \times 26 \times 25 \times 8 \times 7$ in factorial notation.

5. How many arrangements are possible for the word **MERMAID**?

6. What is the seventh element of row twelve of Pascal's Triangle?

7. Determine the fifth term of the expansion of $(3x-4)^8$.

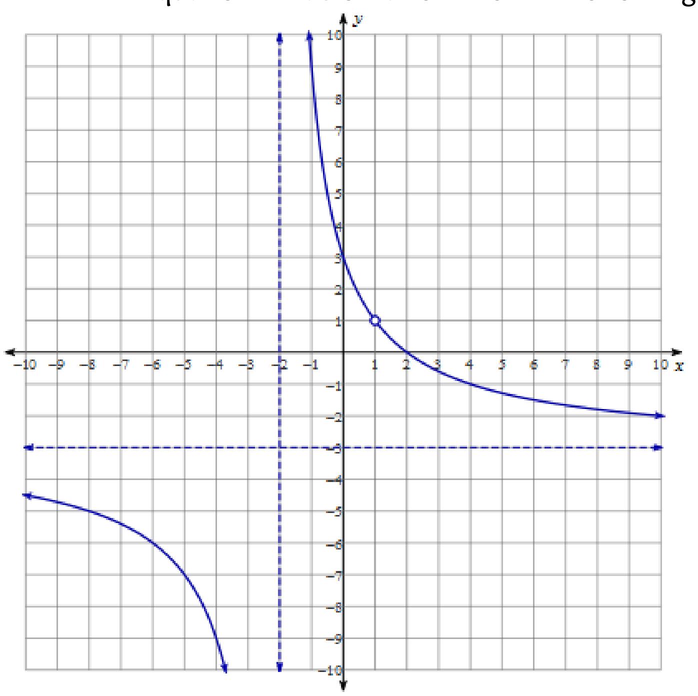
8. Determine the equation in factored form for the following graph.



9. When $4x^3-6x^2+kx-38$ is divided by $x-2$, the remainder is 12. Determine the value of k.

10. Write the equations of any vertical and horizontal asymptotes for the function $y=\dfrac{3x+1}{9x-18}$.

11. Determine the equation in factored form for the following graph.



12. Determine the following limits, if they exist.

a) $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x^2 + 2x - 15}$

b) $\lim_{x \to -\infty} \frac{x^2 + 5}{2x}$

13. In an arithmetic sequence, $t_3 = 3$ and $t_{11} = 51$. What is the common difference?

14. Find the missing terms in the following ***geometric sequence*** :
{4, __, __, 108, …}

15. Find the sum of the following series: $\left\{ \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \dots \right\}$

16. Express $0.747474\dots$ as a fraction.

OPEN RESPONSE:

1. Lunch is made up of an entrée, a side dish, and a drink. The entrées choices are pasta, pizza, or a burger. Side dishes are fries or a salad. Drink choices are milk, pop, or juice. How many unique lunches are possible? Draw a tree diagram to illustrate all possibilities.

2. If 1 9 36 84 126 126 … is part of a row in Pascal's Triangle:

   a) Finish this row
   b) Write the first four elements of the next row.

3. Algebraically solve for n if ${}_{n+1}C_5 = {}_nC_4$.

4. In each of the following, state whether you are dealing with a permutation or a combination, then solve for the number of possibilities.

   a) How many ways can you arrange 6 books on a shelf?
   b) How many ways can you take 4 candies from a jar containing 12?

5. At the local ice-cream parlor, there are 12 flavours of ice-cream and 5 flavours of frozen yogurt from which to choose. If you plan to sample exactly 4 of the 17 flavours before ordering,

   a) How many ways can you sample *exactly* 2 of the ice-cream flavours?
   b) How many ways can you try *at least* 2 of the ice-cream flavours?

6. Use the Binomial Theorem to expand the following expressions.

a) $(2x-3)^5$ b) $(x+4y)^6$ c) $\left(x^3-\dfrac{x^2}{6}\right)^4$

7. Completely factor the following equations.

a. $y=8x^4+27x$

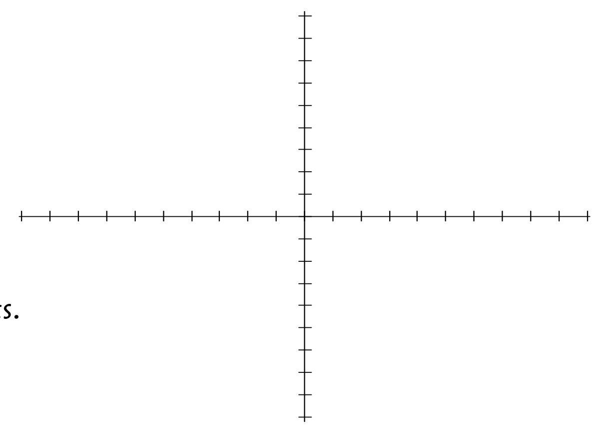
b. $y=2x^4-4x^2-30$

c. $y=x^3+3x^2-16x-48$

d. $y=x^3-7x^2+2x+40$

e. $y=x^4-6x^2+1$

8. For the following function: $y=x^4-19x^2+6x+72$



a) Factor completely.

b) State the x-intercepts of the graph.

c) State the y-intercept of the graph.

d) Make a rough sketch of the graph showing all intercepts.

e) Determine the intervals where the function is positive.

f) Determine the intervals where the function is negative.

9. Rectangular blocks of ice are cut up and used to build the front entrance of an ice castle. The volume, in cubic feet, of each block is represented by $V(x)=5x^3+7x^2-8x-4$, where x is a positive real number. What are the factors, in terms of x, that represent possible dimensions of each block?

10. Perform the division $(x^3+4x^2-7)\div(x+3)$. Express the result in the form $\dfrac{P(x)}{x-a}=Q(x)+\dfrac{R}{x-a}$, and identify any restrictions on the variable.

11. Complete the details, check end-behavior, and then sketch the following.

$$y=\frac{x^3-13x+12}{x^3+x^2-8x-12}=\frac{(x-1)(x-3)(x+4)}{(x+2)(x+2)(x-3)}$$

a) point(s) of discontinuity

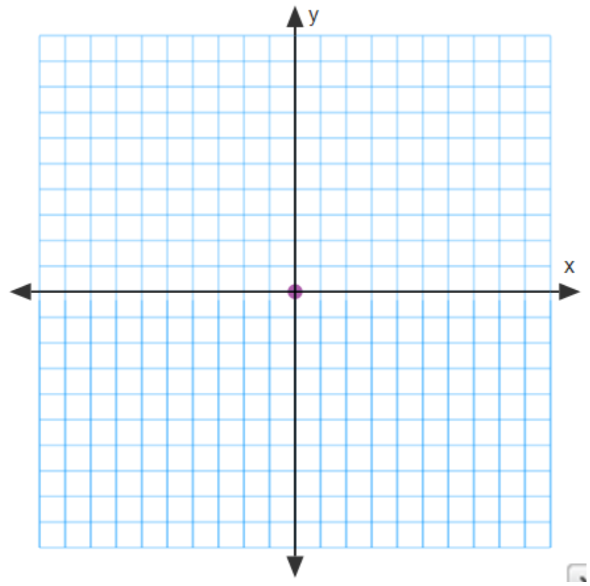
b) x-intercept(s)

c) vertical asymptote(s)

d) y-intercept

e) horizontal asymptote

f) domain

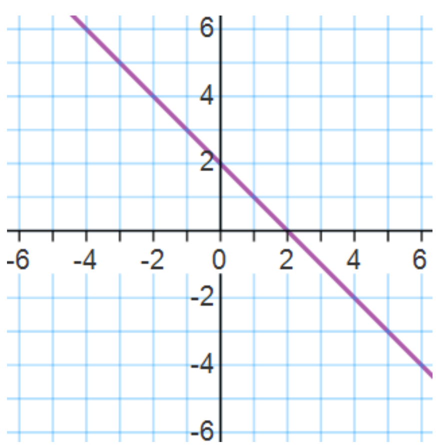


12. Solve for x. Remember to check for extraneous solutions.

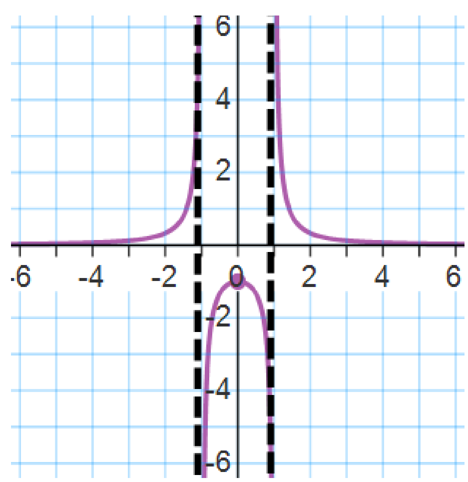
$$\frac{x+4}{x}=\frac{14}{x^2-7x}+\frac{x+3}{x-7}$$

13. Sketch the graph of the reciprocal function for each of the following. Clearly indicate any invariant points.

a)



b)



14. Evaluate the following limits, if they exist.

a. $\lim\limits_{x \to 2} \dfrac{x-2}{x^2-4} =$

b. $\lim\limits_{x \to 0} \dfrac{x^3-6x^2}{x^2} =$

c. $\lim\limits_{x \to 7} \dfrac{x+1}{x-1} =$

d. $\lim\limits_{x \to \infty} \dfrac{x^2-3x+1}{x^3-8} =$

e. $\lim\limits_{x \to \infty} \left(-x^3+8x^2+13\right) =$

f. $\lim\limits_{x \to -\infty} \dfrac{x^4-3x^2-18}{2x^4-32} =$

g. $\lim\limits_{x \to -\infty} 4\left(\dfrac{3}{2}\right)^x =$

h. $\lim\limits_{x \to -3} \sqrt{3x+5} =$

i. $\lim\limits_{x \to 0} \left(2x+\dfrac{x}{8}-5\right) =$

j. $\lim\limits_{x \to -5^+} \begin{Bmatrix} x+7, \ x<-5 \\ \sqrt{2x^2-14}, \ x \geq -5 \end{Bmatrix} =$

15. Given the graph of f(x), find the following:

a. $\lim\limits_{x \to 1^-} f(x) =$

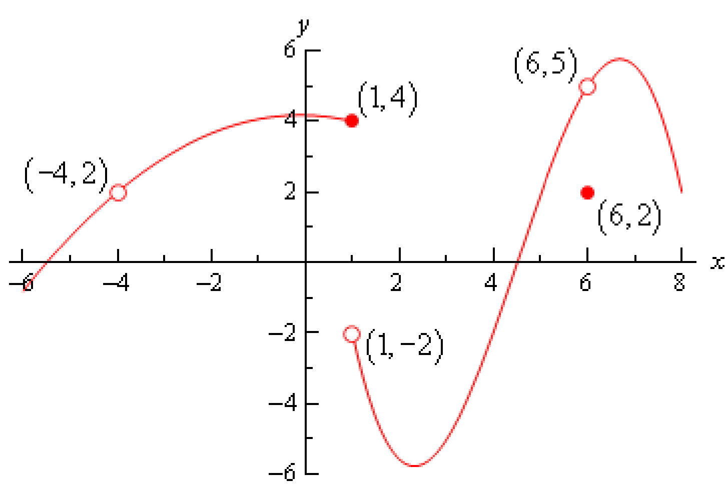
b. $\lim\limits_{x \to 1^+} f(x) =$

c. $\lim\limits_{x \to 1} f(x) =$

d. $\lim\limits_{x \to 6} f(x) =$

e. $f(-4) =$

f. $f(6) =$



16. For what value of b is the following function continuous at $x = 4$?

$$f(x) = \begin{cases} bx + 1 & if\ x \leq 4 \\ bx^2 + 25 & if\ x > 4 \end{cases}$$

17. Find the missing terms for the following sequences.

    a. arithmetic: {__, __, 3, __, __, __, 27, …}

    b. geometric: {__, 3, __, __, __, 48, …}

18. Determine the first term, the common difference and the general term, t_n, for each of the following arithmetic sequences.

    a. $t_{10} = 29$ and $t_{14} = 41$

    b. $t_9 = -6$ and $t_{12} = -12$

19. The sum of the first five terms of a geometric series is 186, and the sum of the first six terms is 378. If the fourth term is 48, find:

    a. the first term
    b. the common ratio
    c. the tenth term
    d. the sum of the first ten terms

20. Evaluate and express the following sums using sigma notation.

    a. $2 + 7 + 12 + \ldots + 57$

    b. $35 + 32 + 29 + \ldots + (-4)$

    c. $1 + 3 + 9 + 27 + \ldots + 6561$

21. The current population of mosquitoes surrounding a lake is 75 000. The population is expected to decrease by 2500 each week after the current week.

    a. Write an expression for the mosquito population in week n.

    b. In which week will the mosquito population reach 40 000?

22. In a movie theatre there are 12 seats in the first row. The next 7 rows increase by 3 seats each and the remaining rows increase by 5 seats. If there are 16 rows in the theatre, how many people can be seated at one time?

23. The Mill has a pile of logs with 34 in the bottom row, 33 logs in the next row, 32 logs in the next row, and so on. If there are 15 logs in the top row, how many logs are there in the pile?

24. This year, Kate bought a vintage Ferrari for $525 000. For the past several years, these models have increased in value by 8% each year.

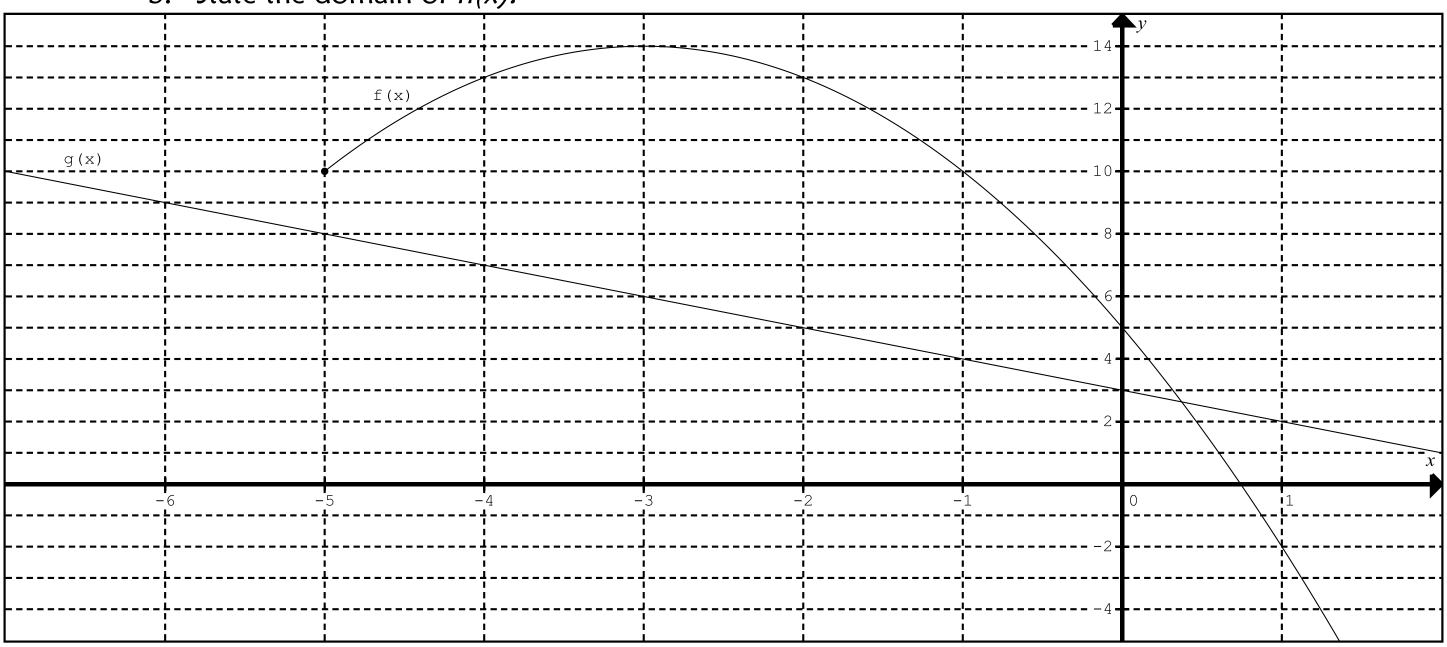
    a. Write an expression for the value of the Ferrari.
    b. What will be the value of her Ferrari in 5 years?
    c. How long will it take for the Ferrari to be worth 1 000 000?

25. In its first year of sap production a maple sugar tree produces 70L of sap. Every year after that sap production decreases by 10%.

    a) How much maple sugar is produced in the first 8 years?
    b) Estimate the tree's total sap production.
    c) When will the sap production drop below 20L per year?

26. Consider the graphs of two functions f(x) and g(x), shown below.

    a. Sketch the graph of $h(x) = f(x) - g(x)$.
    b. State the domain of *h(x).*

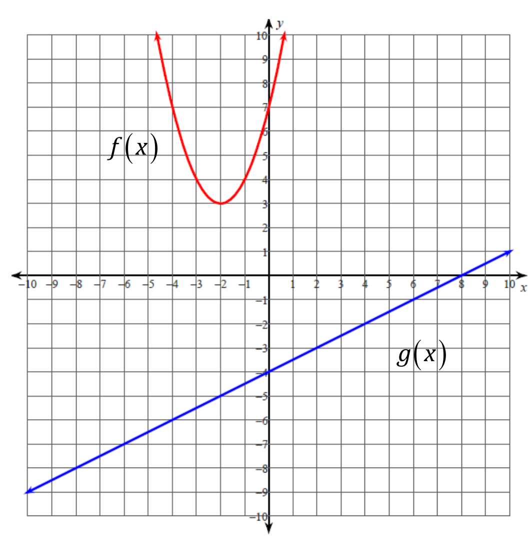


27. Use the graphs of f(x) and g(x) shown to evaluate the following.

    a. $(f \circ g)(2)$

    b. g(g(–2))

    c. $(f \times g)(0)$



28. For $f(x)=5x+1$ and $g(x)=\dfrac{1}{x-4}$, evaluate each of the compositions of functions below.

a. g(f(0)) b. $(g \circ g)(7)$ c. f(g(4)) d. g(f(–5))

29. For $f(x)=x^2+7x+12$ and $g(x)=x-5$ determine the equations of the following.

a. $(f-g)(x)$ b. $f(g(x))$ c. $f(x)\bullet g(x)$

30. Given $f(x)=x^2+5x+6$ and $g(x)=x^2-4$,

a. determine the simplified equation for $h(x)=\dfrac{f(x)}{g(x)}$.

b. state the domain of $h(x)$.

Pre-Calculus 12B Exam Review Solutions

SHORT RESPONSE:

1. $36\times35\times34\times33\times32\times31=1402410240$

2. must end with 5, 7 or 9 to be odd $5\times5\times3=75$

3. $(14\times13\times12)+7=2191$

4. $\dfrac{34!}{31!}\times\dfrac{26!}{24!}\times\dfrac{8!}{6!}$

5. $\dfrac{7!}{2!}=2520$

6. ${}_{12}C_6=924$

7. ${}_8C_4(3x)^4(-4)^4=70(81x^4)(256)=1451520x^4$

8. $y=(x-1)^2(x+3)$

9. $$\begin{aligned}4(2)^3-6(2)^2+k(2)-38&=12\\2k&=42\\k&=21\end{aligned}$$

10. vertical asymptote: x=2 horizontal asymptote: $y=\frac{3}{9}=\frac{1}{3}$

11. $y=\frac{-3(x-1)(x-2)}{(x-1)(x+2)}$

12.

a) $\lim_{x\to 3}\frac{x^2-2x-3}{x^2+2x-15}=\lim_{x\to 3}\frac{(x+1)(x-3)}{(x+5)(x-3)}=\lim_{x\to 3}\frac{(x+1)}{(x+5)}=\frac{4}{8}=\frac{1}{2}$

b) $\lim_{x\to -\infty}\frac{x^2+5}{2x}=-\infty$

13. $d=\frac{51-3}{11-3}=\frac{48}{8}=6$

14. $r^{4-1}=\frac{108}{4}=27$

$r=27^{\frac{1}{3}}=3$

{4, 4x3=12, 12x3=36, 108, ...}

15. $s_\infty=\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$

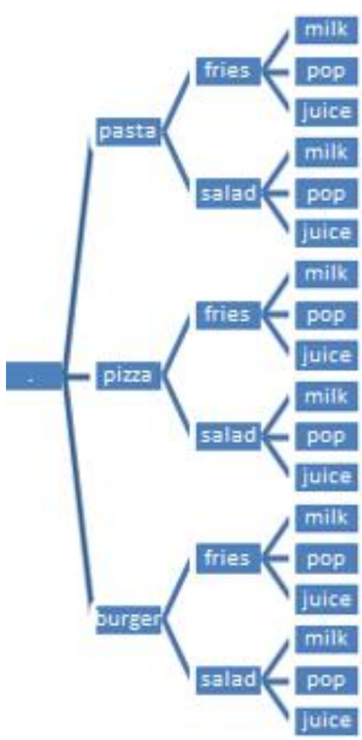
16. $0.747474... = 0.74+0.0074+0.000074+...$

$t_1=0.74,\ r=0.01$

So, $0.747474... = \frac{0.74}{1-.01}=\frac{0.74}{0.99}=\frac{74}{99}$

OPEN RESPONSE:

1. There are 3x2x3 = 18 possible lunch combinations



2. a) 1 9 36 84 126 126 84 36 9 1
   b) 1 10 45 120

$${}_{n+1}C_5 = {}_n C_4$$
$$\frac{(n+1)!}{(n-4)!5!} = \frac{n!}{(n-4)!4!}$$
$$\frac{(n+1)!}{5!} = \frac{n!}{4!}$$

3. $$\frac{(n+1)!}{n!} = \frac{5!}{4!}$$
$$\frac{(n+1)\times n!}{n!} = \frac{5\times 4!}{4!}$$
$$(n+1) = 5$$
$$n = 4$$

4.

a) Permutation $6! = 720$
b) Combination ${}_{12}C_4 = 495$

5. a) ${}_{12}C_2 \times {}_5 C_2 = 66\times 10 = 660$
   b) ${}_{12}C_2 \times {}_5 C_2 + {}_{12} C_3 \times {}_5 C_1 + {}_{12} C_4 = 660 + 1100 + 495 = 2255$

6. a)
$$(2x-3)^5 = {}_5 C_0 (2x)^5 (-3)^0 + {}_5 C_1 (2x)^4 (-3)^1 + {}_5 C_2 (2x)^3 (-3)^2 + {}_5 C_3 (2x)^2 (-3)^3$$
$$+ {}_5 C_4 (2x)^1 (-3)^4 + {}_5 C_5 (2x)^0 (-3)^5$$
$$= 1(32x^5)(1) + 5(16x^4)(-3) + 10(8x^3)(9) + 10(4x^2)(-27) + 5(2x)(81) + 1(1)(-243)$$
$$= 32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243$$

b)
$$(x+4y)^6 = {}_6 C_0 (x)^6 (4y)^0 + {}_6 C_1 (x)^5 (4y)^1 + {}_6 C_2 (x)^4 (4y)^2 + {}_6 C_3 (x)^3 (4y)^3$$
$$+ {}_6 C_4 (x)^2 (4y)^4 + {}_6 C_5 (x)^1 (4y)^5 + {}_6 C_6 (x)^0 (4y)^5$$
$$= 1(x^6)(1) + 6(x^5)(4y) + 15(x^4)(16y^2) + 20(x^3)(64y^3)$$
$$+ 15(x^2)(256y^4) + 6(x)(1024y^5) + 1(1)(4096y^6)$$
$$= x^6 + 24x^5 y + 240x^4 y^2 + 1280x^3 y^3 + 3840x^2 y^4 + 6144xy^5 + 4096y^6$$

c)

$$\left(x^3-\frac{x^2}{6}\right)^4 = {}_4C_0\left(x^3\right)^4\left(-\frac{x^2}{6}\right)^0 + {}_4C_1\left(x^3\right)^3\left(-\frac{x^2}{6}\right)^1 + {}_4C_2\left(x^3\right)^2\left(-\frac{x^2}{6}\right)^2$$

$$+ {}_4C_3\left(x^3\right)^1\left(-\frac{x^2}{6}\right)^3 + {}_4C_4\left(x^3\right)^0\left(-\frac{x^2}{6}\right)^4$$

$$=1\left(x^{12}\right)(1)+4\left(x^9\right)\left(-\frac{x^2}{6}\right)+6\left(x^6\right)\left(\frac{x^4}{36}\right)+4\left(x^3\right)\left(-\frac{x^6}{216}\right)+1(1)\left(\frac{x^8}{1296}\right)$$

$$=x^{12}-\frac{2x^{11}}{3}+\frac{x^{10}}{6}-\frac{x^9}{54}+\frac{x^8}{1296}$$

7.

a.
$$\begin{aligned} y &= 8x^4+27x \\ &= x\left(8x^3+27\right) \\ &= x(2x+3)\left(4x^2-6x+9\right) \end{aligned}$$

b.
$$\begin{aligned} y &= 2x^4-4x^2-30 \\ &= 2\left(x^4-2x^2-15\right) \\ &= 2\left(x^2-5\right)\left(x^2+3\right) \end{aligned}$$

c.
$$\begin{aligned} y &= x^3+3x^2-16x-48 \\ &= x^2(x+3)-16(x+3) \\ &= \left(x^2-16\right)(x+3) \\ &= (x+4)(x-4)(x+3) \end{aligned}$$

d. Requires synthetic or long division.

$$\begin{aligned} y &= x^3-7x^2+2x+40 \\ &= (x+2)(x-4)(x-5) \end{aligned}$$

e.
$$\begin{aligned} y &= x^4 - 6x^2 + 1 \\ &= \left(x^4 - 2x^2 + 1\right) - 4x^2 \\ &= \left(x^2 - 1\right)^2 - 4x^2 \\ &= \left(x^2 - 1 + 2x\right)\left(x^2 - 1 - 2x\right) \end{aligned}$$

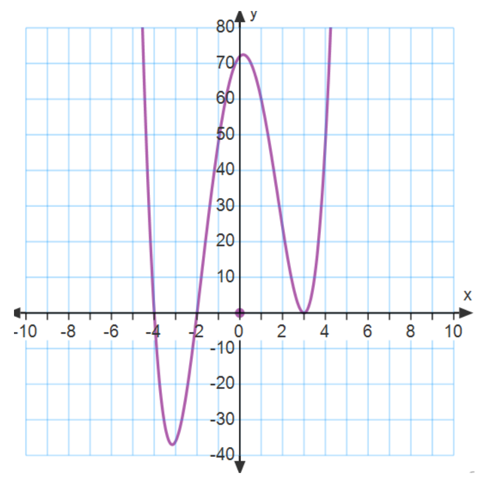
8. $y = x^4 - 19x^2 + 6x + 72$

a) Factor completely: $y = (x-3)^2(x+2)(x+4)$

b) x-intercepts = –4, –2, 3

c) y-intercept = 72

d) sketch of the graph shown to the right



e) positive: $x \in (-\infty, -4) \cup (-2, 3) \cup (3, \infty)$

f) negative: $x \in (-4, -2)$

9. Requires synthetic or long division.

$$\begin{aligned} V(x) &= 5x^3 + 7x^2 - 8x - 4 \\ &= (x-1)(x+2)(5x+2) \end{aligned}$$

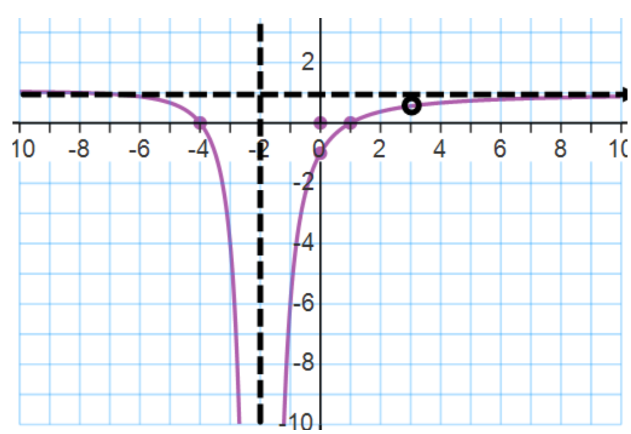
10.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & 0 & -7 \\ & & 3 & 3 & -9 \\ \hline & 1 & 1 & -3 & \lfloor 2 \end{array}$$

$$\therefore \frac{x^3 + 4x^2 - 7}{x+3} = x^2 + x - 3 + \frac{2}{x+3}, \quad x \neq -3$$

11.

$$y = \frac{x^3 - 13x + 12}{x^3 + x^2 - 8x - 12} = \frac{(x-1)(x-3)(x+4)}{(x+2)(x+2)(x-3)}$$



a) POD: $\left(3, \frac{14}{25}\right)$

b) x-intercepts = –4, 1

c) vertical asymptote: x = –2

d) y-intercept = –1

e) horizontal asymptote: y = 1

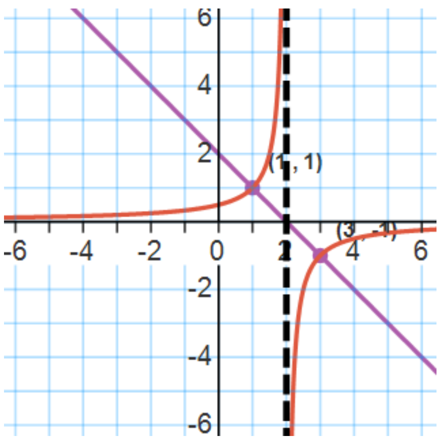
f) domain: $x \in R,\ x \neq -2, 3$

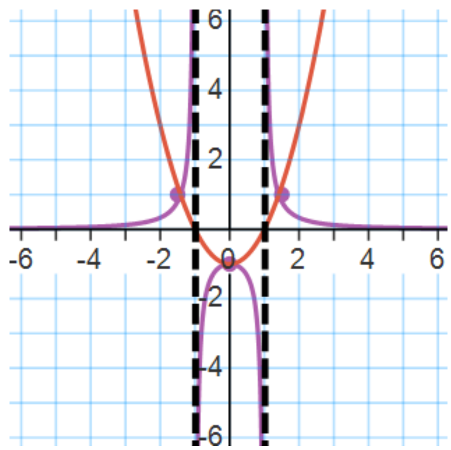
12.

$$x(x-7)\left(\frac{x+4}{x}\right)=x(x-7)\frac{14}{x(x-7)}+x(x-7)\left(\frac{x+3}{x-7}\right)$$
$$x^2-3x-28=14+x^2+3x$$
$$-6x=42$$
$$x=-7$$

13.

a)





14.

a. $\lim\limits_{x\to 2}\frac{x-2}{x^2-4}=\frac{(x-2)}{(x-2)(x+2)}=\frac{1}{2+2}=\frac{1}{4}$

b. $\lim\limits_{x\to 0}\frac{x^3-6x^2}{x^2}=\frac{x^2(x-6)}{x^2}=0-6=-6$

c. $\lim\limits_{x\to 7}\frac{x+1}{x-1}=\frac{8}{6}=\frac{4}{3}$

d. $\lim\limits_{x\to\infty}\frac{x^2-3x+1}{x^3-8}=0$

e. $\lim\limits_{x\to\infty}\left(-x^3+8x^2+13\right)=-\infty$

f. $\lim\limits_{x\to-\infty}\frac{x^4-3x^2-18}{2x^4-32}=\frac{1}{2}$

g. $\lim\limits_{x\to-\infty}4\left(\frac{3}{2}\right)^x=4\left(\frac{3}{2}\right)^{-\infty}=4\left(\frac{2}{3}\right)^{\infty}=0$

h. $\lim\limits_{x\to -3}\sqrt{3x+5}=\sqrt{3(-3)+5}=\sqrt{-9+5}=\sqrt{-4}=DNE$

i. $\lim\limits_{x\to 0}\left(2x+\frac{x}{8}-5\right)=-5$

j. $\lim\limits_{x\to -5^+}\left\{\begin{matrix} x+7,\ x<-5 \\ \sqrt{2x^2-14},\ x\geq -5\end{matrix}\right\}=\lim\limits_{x\to -5}\sqrt{2x^2-14}=\sqrt{2(-5)^2-14}=\sqrt{50-14}=6$

15. Given the graph of f(x), find the following:

a. $\lim\limits_{x\to 1^-} f(x)=4$

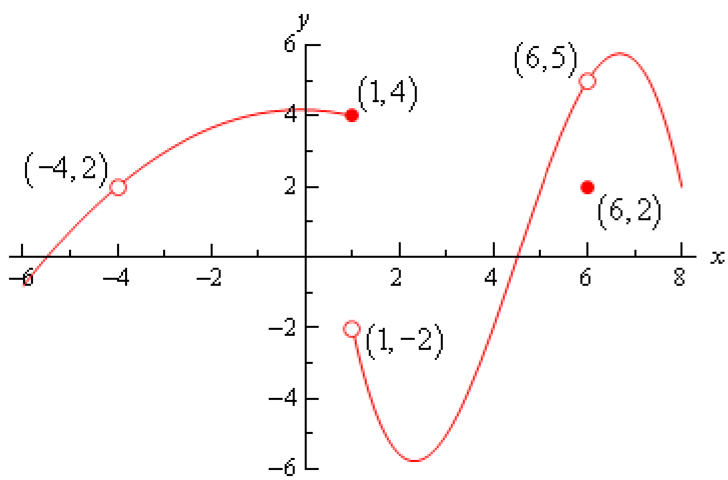
b. $\lim\limits_{x\to 1^+} f(x)=-2$

c. $\lim\limits_{x\to 1} f(x)=DNE$

d. $\lim\limits_{x\to 6} f(x)=5$

e. $f(-4)=undefined$

f. $f(6)=2$



16.

$$4b+1=16b+25$$
$$-24=12b$$
$$-2=b$$

17.

a. arithmetic:

$$d=\frac{27-3}{7-3}=\frac{24}{4}=6$$

$$27=t_1+(7-1)(6)$$
$$27=t_1+36$$
$$-9=t_1$$

$$t_n=-9+(n-1)(6)$$
$$=6n-15$$

{–9, –3, 3, 9, 15, 21, 27, …}

b. geometric:

$$r^{6-2}=\frac{48}{3}=16$$
$$r=(16)^{\frac{1}{4}}=2$$

$$48=t_1(2)^{6-1}$$
$$48=32t_1$$
$$\frac{3}{2}=t_1$$

$$t_n=\frac{3}{2}(2)^{n-1}$$

{3/2, 3, 6, 12, 24, 48, …}

18.

a.

$$d=\frac{41-29}{14-10}=\frac{12}{4}=3$$

$$\begin{aligned}41&=t_1+(14-1)(3)\\41&=t_1+39\\2&=t_1\end{aligned}$$

$$\begin{aligned}t_n&=2+(n-1)(3)\\&=3n-1\end{aligned}$$

b.

$$d=\frac{-12-(-6)}{12-9}=\frac{-6}{3}=-2$$

$$\begin{aligned}-6&=t_1+(9-1)(-2)\\-6&=t_1-16\\10&=t_1\end{aligned}$$

$$\begin{aligned}t_n&=10+(n-1)(-2)\\&=-2n+12\end{aligned}$$

19.

$$\begin{aligned}S_6-S_5&=t_6\\378-186&=192\end{aligned}$$

$$\begin{aligned}r^{6-4}&=\frac{192}{48}=4\\r&=(4)^{\frac{1}{2}}=2\end{aligned}$$

$$\begin{aligned}t_6&=192=t_1(2)^{6-1}\\192&=32t_1\\6&=t_1\end{aligned}$$

$$\begin{aligned}t_{10}&=6(2)^{10-1}\\&=6(512)\\&=3072\end{aligned}$$

$$\begin{aligned}S_{10}&=\frac{6\left(1-2^{10}\right)}{1-2}\\&=\frac{6(-1023)}{-1}\\&=6138\end{aligned}$$

20.

a. 2 + 7 + 12 + … + 57

$$t_1=2,\ d=5$$

$$\begin{aligned}57&=2+(n-1)(5)\\57&=5n-3\\60&=5n\\12&=n\end{aligned}$$

$$\begin{aligned}S_{12}&=\frac{12}{2}(2+57)\\&=6(59)\\&=354\end{aligned}$$

$$\sum_{k=1}^{12}(5n-3)=354$$

b. 35 + 32 + 29 + … + (−4)

$$t_1=35,\ d=-3$$

$$\begin{aligned}-4&=35+(n-1)(-3)\\-4&=-3n+38\\-42&=-3n\\14&=n\end{aligned}$$

$$\begin{aligned}S_{14}&=\frac{14}{2}(35+(-4))\\&=7(31)\\&=217\end{aligned}$$

$$\sum_{k=1}^{14}(-3n+38)=217$$

c. 1 + 3 + 9 + 27 + … + 6561

$$t_1 = 1,\ r = 3$$

$$6561 = 1(3)^{n-1}$$
$$n - 1 = \frac{\log(6561)}{\log(3)} = 8$$
$$n = 9$$

$$S_9 = \frac{1\left(1-3^9\right)}{1-3}$$
$$= \frac{(-19682)}{-2}$$
$$= 9841$$

$$\sum_{k=1}^{9} 1(3)^{n-1} = 9841$$

21.

a. $t_1 = 75000,\ d = -2500$

$$t_n = 75000 + (n-1)(-2500)$$
$$= -2500n + 77500$$

b.
$$40000 = -2500n + 77500$$
$$-37500 = -2500n$$
$$15^{th}\ week = n$$

22. There are 2 patterns here, the first eight rows and the last eight rows.

First eight rows:

$$t_1 = 12,\ d = 3$$
$$S_8 = \frac{8}{2}\left(2(12) + (8-1)(3)\right)$$
$$= 4\left(24 + 7(3)\right)$$
$$= 4(45) = 180\ seats$$

Last eight rows:

$$t_8 = 12 + (8-1)(3) = 33$$
$$\therefore t_9 = 33 + 5 = 38$$
$$\therefore t_1 = 38,\ d = 5$$

$$S_8 = \frac{8}{2}\left(2(38) + (8-1)(5)\right)$$
$$= 4\left(76 + 7(5)\right)$$
$$= 4(111) = 444\ seats$$

Therefore the total number of seats is 180+444=624 seats.

Or expand the series: 12+15+18+21+24+27+30+33+38+43+48+53+58+63+68+73=624 seats

23.

$t_1 = 34,\ d = -1$

$$\begin{aligned}15 &= 34 + (n-1)(-1)\\ 15 &= -n + 35\\ -20 &= -n\\ 20 &= n\end{aligned}$$

$$\begin{aligned}S_{20} &= \frac{20}{2}(34+15)\\ &= 10(49)\\ &= 490 \text{ logs}\end{aligned}$$

24.

a. $t_1 = 525000,\ r = 108\% = 1.08$ $\quad t_n = 525000(1.08)^{n-1}$

b.

$$\begin{aligned}t_6 &= 525\ 000(1.08)^{6-1}\\ &= 525\ 000(1.4693...)\\ &= \$771\ 397.24\end{aligned}$$

c.

$$\begin{aligned}1000000 &= 525000(1.08)^{n-1}\\ 1.90476... &= (1.08)^{n-1}\\ n-1 &= \frac{\log(1.90476...)}{\log(1.08)} = 8.3725...\end{aligned}$$

So, in 9 years.

25.

a. $t_1 = 70,\ r = 90\% = 0.9$

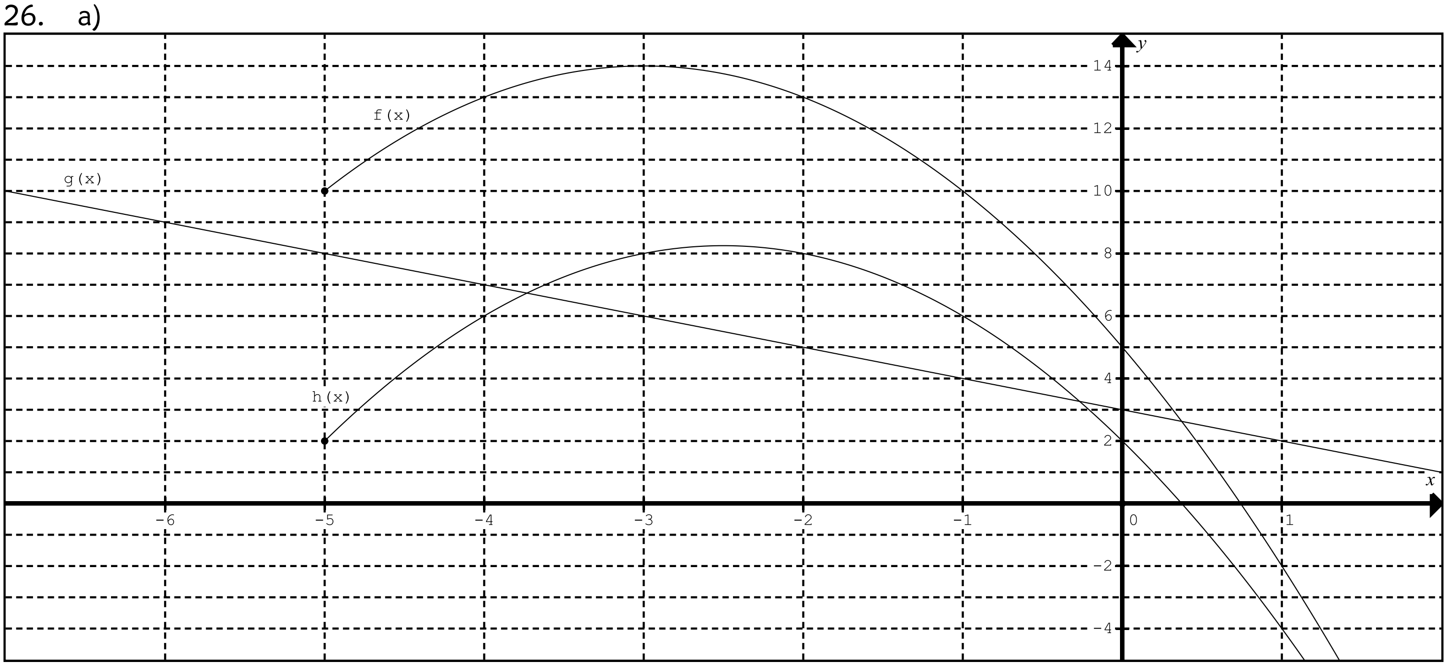
$$\begin{aligned}S_8 &= \frac{70(1-0.9^8)}{1-0.9}\\ &= \frac{70(0.5696...)}{0.1}\\ &= 398.67\,L\end{aligned}$$

b. $S_\infty = \dfrac{70}{1-0.9} = 700\,L$

$$c.\ \ 20 > 70(0.9)^{n-1}$$
$$\frac{2}{7} > (0.9)^{n-1}$$
$$\log\left(\frac{2}{7}\right) > \log(0.9)^{n-1}$$
$$\frac{\log\left(\frac{2}{7}\right)}{\log(0.9)} < n-1$$
$$11.89 < n-1$$
$$12.89 < n$$

Therefore, in the 13th year (or, in 12 years).

26. a)



b) Domain of h(x): $x \in [-5, \infty)$

27.

a) $(f \circ g)(2) = f(g(2)) = f(-3) = 4$

b) $g(g(-2)) = g(-5) = -6.5$

c) $(f \times g)(0) = f(0)g(0) = (7)(-4) = -28$

28. $f(x)=5x+1 \qquad g(x)=\dfrac{1}{x-4}$

a)

$$g(f(0))=g(1)=-\frac{1}{3}$$

b)

$$(g\circ g)(7)=g(g(7))=g\left(\frac{1}{3}\right)=\frac{1}{\frac{1}{3}-4}=\frac{1}{\left(-\frac{11}{3}\right)}=-\frac{3}{11}$$

c)

$$f(g(4))=f\left(\frac{1}{0}\right)=undefined$$

d)

$$g(f(-5))=g(-24)=-\frac{1}{28}$$

29. $f(x)=x^2+7x+12 \qquad g(x)=x-5$

a)

$$\begin{aligned}(f-g)(x)&=x^2+7x+12-(x-5)\\&=x^2+7x+12-x+5\\&=x^2+6x+17\end{aligned}$$

b)

$$\begin{aligned}f(g(x))=f(x-5)&=(x-5)^2+7(x-5)+12\\&=x^2-10x+25+7x-35+12\\&=x^2-3x+2\end{aligned}$$

c)

$$\begin{aligned}f(x)g(x)&=(x^2+7x+12)(x-5)\\&=x^3+7x^2+12x-5x^2-35x-60\\&=x^3+2x^2-23x-60\end{aligned}$$

30.

$f(x)=x^2+5x+6 \qquad g(x)=x^2-4$

a) $h(x)=\dfrac{f(x)}{g(x)}=\dfrac{x^2+5x+6}{x^2-4}=\dfrac{(x+3)(x+2)}{(x-2)(x+2)}=\dfrac{x+3}{x-2}$

b) $D:\{x\in R, x\neq \pm 2\}$