

## Rational Functions

**Rational functions** are those that can be written in the form  $f(x) = \frac{n(x)}{d(x)}$ , where  $n(x)$  and  $d(x)$  are both \_\_\_\_\_.

**Skill #1:** Convert functions in *transformation* form  $\left(f(x) = \frac{a}{x-h} + k\right)$  to *rational* form  $\left(f(x) = \frac{n(x)}{d(x)}\right)$ .

The process is like changing a mixed number into an improper fraction.

$$\begin{array}{c} \text{Add} \\ \text{Multiply} \end{array} \quad 5\frac{2}{3} = \frac{5 \times 3 + 2}{3} = \frac{17}{3}$$

Multiply, then add  
(Leave the denominator alone)

**Example:**

Write  $f(x) = 5 + \frac{2}{x-3}$  in the form  $f(x) = \frac{n(x)}{d(x)}$ .

Multiply the whole number (5) and the denominator ( $x-3$ ). Then add the numerator (2). Leave the denominator alone.

$$f(x) = 5 + \frac{2}{x-3} = \frac{5(x-3) + 2}{x-3} = \frac{5x - 15 + 2}{x-3} = \frac{5x - 13}{x-3}$$

**Note:** This process works because it is a shortcut for finding a common denominator between the two terms.

$$5 + \frac{2}{x-3} = \frac{5}{1} + \frac{2}{x-3} = \frac{5(x-3)}{x-3} + \frac{2}{x-3} = \frac{5(x-3) + 2}{x-3}$$

Rewrite each function in the form  $f(x) = \frac{n(x)}{d(x)}$ .

1)  $f(x) = \frac{5}{x-8} + 2$

4)  $f(x) = \frac{1}{2x-6} - 5$

2)  $f(x) = \frac{-3}{x-4} + 1$

5)  $f(x) = \frac{4}{(x-1)^2} + 6$

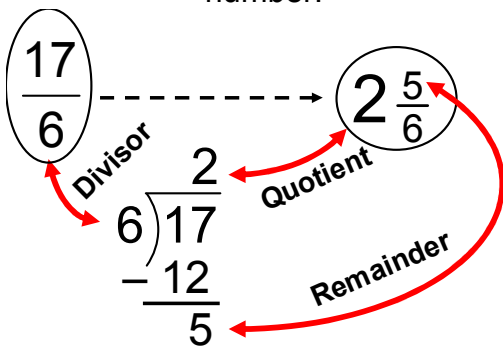
3)  $f(x) = \frac{9}{x+7} - 3$

6)  $f(x) = \frac{3}{(x-5)^2} - 2$

## Rational Functions

**Skill #2:** Convert functions from *rational* form  $\left(f(x) = \frac{n(x)}{d(x)}\right)$  to *transformation* form  $\left(f(x) = \frac{a}{x-h} + k\right)$ .

The process is like changing an improper fraction into a mixed number.



(Divide, and leave the remainder over the original denominator.)

**Example:**

Write  $f(x) = \frac{7x+6}{x+1}$  in the form  $f(x) = \frac{a}{x-h} + k$ .

$$\begin{array}{r} 7 \text{ ← Quotient} \\ x+1 \overline{) 7x+6} \\ \underline{-(7x+7)} \\ -1 \text{ ← Remainder} \end{array}$$

$$f(x) = \underbrace{7}_{\text{Quotient}} + \frac{-1}{x+1}$$

Divide using long division. The answer is the quotient, plus the remainder over the original divisor (or, the denominator).

**Note:**

The answer can also be written as  $f(x) = \frac{-1}{x+1} + 7$

Use long division to rewrite each function in the form  $f(x) = \frac{a}{x-h} + k$ .

7)  $f(x) = \frac{4x+9}{x+2}$

8)  $f(x) = \frac{-7x+38}{x-5}$

9)  $f(x) = \frac{2x-22}{x-8}$

10)  $f(x) = \frac{-9x-4}{x+1}$

Use long division, then rewrite each function in the form  $f(x) = \frac{a}{(x-h)^2} + k$ .

11)  $f(x) = \frac{2x^2+12x+23}{x^2+6x+9}$

12)  $f(x) = \frac{-3x^2+24x-54}{x^2-8x+16}$