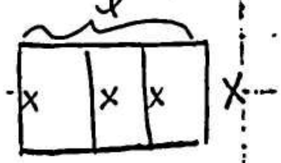
Review Questions - Solutions

p177. #17 a, c, d

a)



280m of fencing

x = width

$2l + 4x = 280\text{m}$

$\frac{\cancel{2}l}{\cancel{2}} = \frac{280\text{m}}{2} - \frac{4x}{2}$

$l = 140\text{m} - 2x$

Area $= l \cdot x$

$A = (140 - 2x)(x)$

$A = 140x - 2x^2$

$A = -2x^2 + 140x$ (Quadratic → polynomial of degree two.)

c) Solve either by completing the square $\underline{\underline{\text{or}}}$ using formulas $\left(p = -\frac{b}{2a}\right)$

$A = -2(x^2 - 70x)$

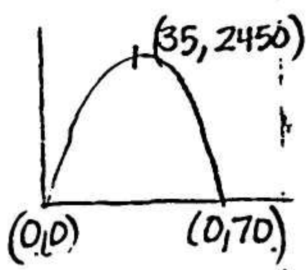
$A = -2(x^2 - 70x + 1225 - 1225)$

$A = -2(x - 35)^2 + 2450$

The vertex is at (35, 2450)

They represent the width ($x = 35$m) when the area is at it's max value ($A = 2450\text{m}^2$.)

d.) Domain: $\{x \mid 0 \leq x \leq 70, x \in R\}$



$A = -2x^2 + 140$

$A = -2x(x - 70)$

$0 = -2x(x - 70)$

$x = 0$, $x = +70$

Range: $\{A \mid 0 \leq A \leq 2450, A \in R\}$

Review Questions – Solutions

p195 # 18 ab

Revenue = (ticket price)(# of tickets sold)

let x = # of \$1.00 decreases in ticket price

$R(x) = (70 - x)(2000 + 50x)$

$R(x) = 140000 + 3500x - 2000x - 50x^2$

$R(x) = \underset{a}{-50}x^2 + \underset{b}{1500}x + \underset{c}{140000}$

Find Max R(x) ⇒ either complete the square to find vertex form or use eqns to find vertex.

(p, q)

$p = \frac{-b}{2a}$ $\qquad$ $q = c - \frac{b^2}{4a}$

$p = \frac{-1500}{2(-50)}$ $\qquad$ $q = 140000 - \frac{(1500)^2}{4(-50)}$

$p = 15$ $\qquad$ $q = 151250$

↑ x (# of \$1.00 decreases) $\qquad$ ↑ R

Ticket price = 70 − x
= 70 − 15
= \$55

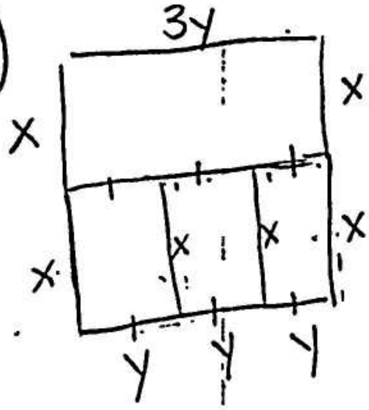
Revenue = \$151250

The max. revenue is \$151250.00 when the ticket price is \$55.00

b) # of tickets sold at \$55.00 (when x = 15) = (2000 + 50x)
= 2000 + 50(15)
= 2750 tickets.

p196 #22, #23, #24

#22)



$$\text{Fencing} = 3y + 2x + 2x + 2x + 3y + 3y$$

$$\text{Fencing} = 9y + 6x$$

$$900 = 9y + 6x$$

$$\frac{900 - 9y}{6} = \frac{\cancel{6}x}{\cancel{6}}$$

$$150 - 1.5y = x$$

$$\text{Area} = 2x \cdot 3y$$

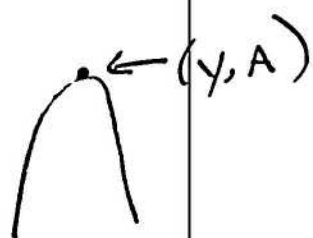
$$A = 2(150 - 1.5y) \cdot 3y$$

$$A = (300 - 3y) \cdot 3y$$

$$A = 900y - 9y^2$$

$$A = -9y^2 + 900y$$

Maximum (find vertex!)
Area?



$$p = \frac{-b}{2a} \qquad q = c - \frac{b^2}{4a}$$

$$p = \frac{-900}{2(-9)} \qquad = 0 - \frac{(900^2)}{4(-9)}$$

$$p = 50 \qquad q = 22500$$

y = 50 — y value @ max. area.

22500 — max area

find x:

$$150 - 1.5y = x$$

$$150 - 1.5(50) = x$$

x = 75.

When x = 75 and y = 50, the area is maximized.
(Large fields: 75 by 150m)
small: (75 by 50m)

#23 a) $n+m=29$ $\quad$ $n \cdot m = \text{Product}$

$29-m=n$ $\quad$ $(29-m)(m) = \text{Product}$

$29m - m^2 = \text{Product}$

max @vertex

$\text{Product} = -m^2 + 29m$

$\Longrightarrow$ Find vertex (p, q)

$p = \frac{-b}{2a}$ $\quad$ $q = c - \frac{b^2}{4a}$

$p = -\frac{29}{2(-1)}$ $\quad$ $q = 0 - \frac{(29)^2}{4(-1)}$

$p = 14.5$ $\quad$ $q = 210.25$

m at vertex.

$\boxed{\begin{array}{c}\text{Max product} = 210.25 \\ m = 14.5 \\ n = 14.5\end{array}}$

$n = 29-m$

$n = 29-14.5$

$n = 14.5$

23 b) $n-m=13$ $\quad$ $13+m=n$

Vertex: $(-6.5, -42.25)$

$\boxed{\text{Min Product} = -42.25}$

when $\boxed{m = -6.5}$

$n-(-6.5)=13$

$n+6.5=13$

$\boxed{n=6.5}$

$n \cdot m = \text{product}$

$(13+m)(m) = \text{Product}$

$m^2 + 13m = \text{Product}$

min @ vertex.

$\left(m^2 + 13m + \frac{169}{4} - \frac{169}{4}\right) = \text{Prod.}$

$(m+6.5)^2 - \frac{169}{4} = \text{Product}$

$(m+6.5)^2 - 42.25 = \text{Product}$

#24 p. 196

$450 = 3l + 4w$

$450 - 4w = 3l$

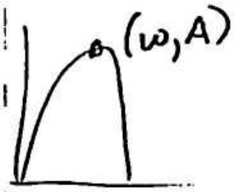
$150 - \frac{4}{3}w = l$

$A = 2w \cdot l$

$A = 2w\left(150 - \frac{4}{3}w\right)$

$A = 300w - \frac{8}{3}w^2$

$A = -\frac{8}{3}w^2 + 300w$



Find vertex:

$p = \frac{-b}{2a}$ $\quad q = c - \frac{b^2}{4a}$

$p = \frac{-300}{2\left(-\frac{8}{3}\right)}$ $\quad q = 0 - \frac{(300^2)}{4(-8/3)}$

$p = 56.25$ $\quad q = 8437.5$

The maximum area is $8437.5\,cm^2$.

p 200 #17abde

#17.

let x = # of \$2.00 decrease in price.

a) Revenue = (price) * (# of sales)

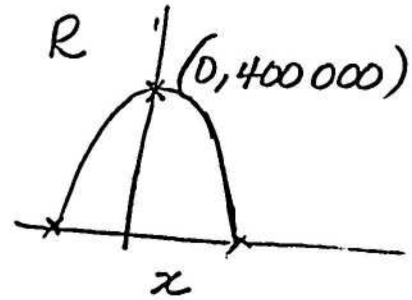
$$R = (40-2x)(10\,000 + 500x)$$

$$R = 400\,000 + \cancel{20\,000x} - \cancel{20\,000x} - 1000x^2$$

$$R = -1000x^2 + 400\,000$$

b) ($\Rightarrow$ can complete the square or use formulas)

$$R = -1000x^2 + 400000$$



$p = \frac{-b}{2a}$ $\quad q = c - \frac{b^2}{4a}$

$p = 0$ $\quad q = 400\,000 - 0$

$q = 400\,000$

The maximum revenue is \$400 000 when there have been no decreases in price, ie: the price is \$40.00 per coat.

$x = 0$

price $= (40-2x)$

price $= 40 - 2(0)$

price $= 40$

c) (on calculator.)

d) Y-intercept represents the revenue before you change the price.

x intercepts show the # of price changes (↑ or ↓) that will still provide revenue.

e) domain: $\{x \mid -20 \leq x \leq 20, x \in R\}$

range: $\{y \mid 0 \leq y \leq 400\,000, y \in R\}$

$$0 = -1000x^2 + 400000$$

$$\frac{-400000}{-1000} = x^2$$

$$\sqrt{400} = x$$

$x = 20$

$x = -20$

p 202 #12a.

$C(v) = 0.004v^2 - 0.62v + 30$

Most efficient speed = v at minimum consumption

$\therefore$ find vertex. $\Rightarrow p$

$p = \dfrac{-b}{2a}$

$p = \dfrac{-(-0.62)}{2(0.004)}$

$p = 77.5$

(for interest,)

$q = c - \dfrac{b^2}{4a}$

$q = 30 - \dfrac{(-0.62)^2}{4(0.004)}$

$q = 5.975 \text{ L}/100 \text{ km}$

The most efficient speed would be $77.5 \frac{km}{h}$.