Homework solutions:

p.452 # 9, 10, 11, 13, 16, 3e, 4bd, 5bc

p.452 #9)

a) perimeter:

$5x-1+2x+y+14=60$

$7x+y-47=0$

b) area:

$\frac{\cancel{2}x(5x-1)}{\cancel{2}} = 10y$

$5x^2-x=10y$

$5x^2-x-10y=0$

c) ① $7x+y-47=0$

② $5x^2-x-10y=0$

The solution(s) to this system will give the values of x and y that give the correct area and perimeter simultaneously.

d) $10\times①$: $70x+10y-470=0$

$+$ ②: $5x^2-x-10y=0$

$5x^2+69x-470=0$

$x = \frac{-69 \pm \sqrt{14161}}{10}$

$= \frac{-69 \pm 119}{10} = 5, -18.8$ ← inadmissible

Find y: ① $7x+y-47=0$

$7(5)+y-47=0$

$y=12$

$\therefore x=5$, so base $=5x-1=5(5)-1=\underline{24m}$

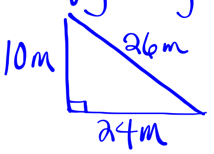
height $=2x=2(5)=\underline{10m}$

hypotenuse $=y+14=12+14$

$=\underline{26m}$

e) Verify:

Pythagorean Theorem:



$10^2+24^2=26^2$?

$100+576=676$

yes.

10) a) Let a represent the larger integer
let b represent the smaller integer

① $b - a = -30$

② $a + 3 + b^2 = 189$

b) ① $-a + b + 30 = 0$
$+$ ② $a + b^2 - 186 = 0$

$$b^2 + b - 156 = 0$$
$$(b+13)(b-12) = 0$$
$$b = -13, 12$$

Find a:

If $b = -13$, ① $b - a = -30$
$-13 - a = -30$
$-13 + 30 = a$
$17 = a$

If $b = 12$, ① $b - a = -30$
$12 - a = -30$
$12 + 30 = a$
$42 = a$

$\therefore$ The integers are -13 and 17
or 12 and 42

c) Verify:

① Two integers have a difference of -30

$-13 - 17 = -30$ ✓

$12 - 42 = -30$ ✓

② When the larger integer is increased by 3 and added to the square of the smaller integer, the result is 189.

$(17+3) + (-13)^2 = 189$ ✓

$(42+3) + (12)^2 = 189$ ✓

11) a)

Number of cm in circumference, $N = 2\pi r$

Three times the # of cm^2 in area, $N = 3\pi r^2$

① $N = 2\pi r$

② $N = 3\pi r^2$

b) $2\pi r = 3\pi r^2$

$0 = 3\pi r^2 - 2\pi r$

$0 = \pi r(3r - 2)$

$r = 0, \frac{2}{3}$

inadmissible

$\therefore$ radius $= \frac{2}{3}$ cm

Area $= \pi r^2 = \pi\left(\frac{2}{3}\right)^2 = \frac{4\pi}{9}$ cm^2

Circumference $= 2\pi r = 2\pi\left(\frac{2}{3}\right) = \frac{4\pi}{3}$ cm

Verify:

# of cm in circumference is triple the # of cm^2 in area:

$\frac{4\pi}{3} = 3\left(\frac{4\pi}{9}\right)$ ✓

13) ① $h(t) = -4.9t^2 + 2015$

② $h(t) = -10.5t + 980$

a) $-4.9t^2 + 2015 = -10.5t + 980$

$0 = 4.9t^2 - 10.5t - 1035$

$$t = \frac{10.5 \pm \sqrt{20396.25}}{9.8}$$

$\doteq +15.64, -13.50$

$\therefore$ The stuntman falls for $\underline{\underline{15.64 s}}$ before opening his parachute.

b) ① $h(15.64) = -4.9(15.64)^2 + 2015$

$\doteq 816 m$

c) verify using equation ②:

$h(15.64) = -10.5(15.64) + 980$

$\doteq 816 m$

$\therefore$ The stuntman was approx. $\underline{\underline{816 m}}$ above the ground when he opened the parachute.

16) a) The solution(s) to the system will give the horizontal distance from the base of the mountain and the height of the explosive (from the base) on the mountain

b) $$-\frac{5}{1600}x^2 + 200 = 1.19x$$

$$5x^2 - 320000 = -1904x$$

$$5x^2 + 1904x - 320000 = 0$$

$$x = \frac{-1904 \pm \sqrt{10\,025\,216}}{10}$$

$$\doteq 126.23, -507.23$$

inadmissible

① $h = 1.19x = 1.19(126.23) \doteq 150 \text{ m}$

verify in ②: $h = -\frac{5}{1600}(126.23)^2 + 200 \doteq 150\text{m}$

∴ The explosive charge lands $\underline{\underline{150\text{m}}}$ up the mountain.

3e) ① $y + 2x = x^2 - 6$

② $x + y - 3 = 2x^2$

① $0 = x^2 - 2x - y - 6$

② $-(0 = 2x^2 - x - y + 3 = 0)$

$$0 = -x^2 - x - 9 = 0$$

$$0 = x^2 + x + 9 = 0$$

$$x = \frac{-1 \pm \sqrt{-35}}{2}$$ ← non-real (complex) numbers

∴ No solution exists

(btw, if you graph the system, you'll see that the two graphs do not intersect)

4b) $x^2 + y = 8x + 19$

$+ \ (x^2 - y = 7x - 11)$

$2x^2 = 15x + 8$

$2x^2 - 15x - 8 = 0$

$2x^2 - 16x + x - 8 = 0$

$2x(x-8) + 1(x-8) = 0$

$(x-8)(2x+1) = 0$

$x = 8, -\frac{1}{2}$

Find y:

If $x = 8$, ① $64 + y = 64 + 19$

$y = 19$

If $x = -\frac{1}{2}$, ① $\frac{1}{4} + y = -4 + 19$

$y = 15 - \frac{1}{4} = \frac{59}{4}$

$\therefore$ The solutions are $(8, 19)$ + $\left(-\frac{1}{2}, \frac{59}{4}\right)$

(verify these using equation ②...)

4d) ① $9w^2 + 8k = -14$
② $w^2 + k = -2$

① $9w^2 + 8k = -14$
$-8 \times$ ② $(-8w^2 - 8k = 16)$
+ ________________
$w^2 = 2$
$w = \pm\sqrt{2}$

Find k:
If $w = \sqrt{2}$, ② $(\sqrt{2})^2 + k = -2$
$k = -2 - 2 = -4$

If $w = -\sqrt{2}$, $k = -4$

Solutions are $(\sqrt{2}, -4)$ & $(-\sqrt{2}, -4)$

(Verify these using equation ①...)

5b) ① $8x^2 + 5y = 100$

② $6x^2 - x - 3y = 5$

$3 \times ①: \quad 24x^2 + 15y = 300$

$+ \; 5 \times ②: \quad (30x^2 - 5x - 15y = 25)$

$54x^2 - 5x = 325$

$54x^2 - 5x - 325 = 0$

$$x = \frac{5 \pm \sqrt{70225}}{108}$$

$$= \frac{5 \pm 265}{108}$$

$$= \frac{270}{108}, \frac{-260}{108}$$

$$= \frac{5}{2}, \frac{-65}{27}$$

Find y:

If $x = \frac{5}{2}$, ① $5y = -8x^2 + 100$

$5y = -8\left(\frac{25}{4}\right) + 100$

$5y = 50$

$y = 10$

If $x = \frac{-65}{27}$, ① $5y = -8x^2 + 100$

$5y = -8\left(\frac{4225}{729}\right) + 100$

$5y = \frac{-33800}{729} + 100$

$5y = \frac{39100}{729}$

$y = \frac{7820}{729}$

∴ The solutions are:

$\left(\frac{5}{2}, 10\right)$ and $\left(\frac{-65}{27}, \frac{7820}{729}\right)$

(Verify these using equation ②...)

5c) ① $x^2 - \frac{48}{9}x + \frac{1}{3}y + \frac{1}{3} = 0$

② $-\frac{5}{4}x^2 - \frac{3}{2}x + \frac{1}{4}y - \frac{1}{2} = 0$

$9\times$①: $9x^2 - 48x + 3y + 3 = 0$

$-12\times$②: $(15x^2 + 18x - 3y + 6 = 0)$

$+$

$24x^2 - 30x + 9 = 0$

$8x^2 - 10x + 3 = 0$

$8x^2 - 6x - 4x + 3 = 0$

$2x(4x-3) - 1(4x-3) = 0$

$(4x-3)(2x-1) = 0$

$x = \frac{3}{4}, \frac{1}{2}$

Find y:

If $x = \frac{3}{4}$, ①: $0.5625 - 4 + \frac{1}{3}y + 0.\overline{3} = 0$

$\frac{1}{3}y = +3.1041\overline{6}$

$y = +9.3125$

If $x = \frac{1}{2}$, ① $0.25 - 2.\overline{6} + \frac{1}{3}y + 0.\overline{3} = 0$

$\frac{1}{3}y = 2.08\overline{3}$

$y = 6.25$

The solutions are:

$(0.75, 9.3125)$ and $(0.5, 6.25)$

(Verify these using equation ②...)