Worksheet – The Ambiguous Case of the Sine Law

1. Determine whether each description of a triangle involves the *SSA* situation.

a) In $\triangle ABC$, $\angle A = 13°$, $a = 2$ cm, and $b = 6$ cm.

b) In $\triangle DEF$, $\angle D = 89°$, $d = 14$ cm, and $f = 11$ cm.

c) In $\triangle PQR$, $\angle P = 38°$, $q = 27$ cm, and $r = 19$ cm.

2. For each description in question 1 that involves the *SSA* situation, determine whether zero, one, or two triangles are possible. Give your reasons.

3. Calculate the height of each triangle, to the nearest tenth of a centimetre. Determine the number of triangles that are possible. Give your reasons.

a) In $\triangle RST$, $\angle R = 103°$, $r = 16$ cm, and $s = 9$ cm.

b) In $\triangle XYZ$, $\angle X = 50°$, $x = 5.2$ cm, and $z = 7.1$ cm.

c) In $\triangle ABC$, $\angle A = 74°$, $a = 28.0$ cm, and $b = 28.9$ cm.

4. Given each set of measurements for $\triangle ABC$, determine the number of triangles that are possible. Draw a diagram to support your answer.

a) $\angle A = 110°$, $a = 4.7$ cm, and $b = 5.1$ cm

b) $\angle A = 50°$, $a = 6.3$ cm, and $b = 8.2$ cm

5. In $\triangle XYZ$, $\angle X = 67°$, $x = 3.2$ m, and $y = 3.4$ m.

a) Determine a possible measure for $\angle Y$, to the nearest degree.

b) State another possible value for $\angle Y$.

6. In $\triangle MNP$, $n = 4.5$ cm and $\angle M = 35°$.

What is the height of the triangle from base p?

A. 4.5 cm **B.** 3.7 cm **C.** 2.6 cm **D.** 0.4 cm

7. In $\triangle PQR$, $\angle P = 108°$, $q = 4.9$ m, and $p = 4.5$ m.

Which statement is true for this set of measurements?

A. This is not an *SSA* situation.

B. This is an *SSA* situation; no triangle is possible.

C. This is an *SSA* situation; only one triangle is possible.

D. This is an *SSA* situation; two triangles are possible.

8. Which set of measurements can produce two possible triangles?

A. $\angle A = 62°$, $a = 3.2$ m, $b = 4.0$ m

B. $\angle A = 52°$, $a = 6.8$ m, $b = 7.4$ m

C. $\angle A = 102°$, $a = 6.2$ m, $b = 9.0$ m

D. none of these

9. In $\triangle ABC$, $\angle B = 24°$, $b = 18$ cm, and $a = 22$ cm. Calculate the measure of $\angle A$, to the nearest degree. Is there more than one possible answer? Explain how you know.

10. In $\triangle DEF$, $\angle D = 97°$, $d = 5.3$ cm, and $e = 4.8$ cm. Calculate the measure of $\angle E$, to the nearest degree. Is there more than one possible answer? Explain how you know.

11. Gail works as an aerial photographer. On one trip she takes off from the airport and flies for 52 km on a bearing of N50°E. Then she turns and flies southwest for 38 km, until she is due east of the airport.

a) Sketch a diagram of Gail's flight.

b) Explain how this situation could be ambiguous, and why it is not.

c) How far is Gail from the airport when she is due east of it?

12. In ΔJKM, $\angle J = 65^{o}$, j = 8.7 m, and k = 9.3 m.

a. Determine how many triangles are possible given these measurements. *Clearly* show how you know.
b. Calculate the length of side JK for each case in (a).
c. Calculate the area of the triangle in each case.

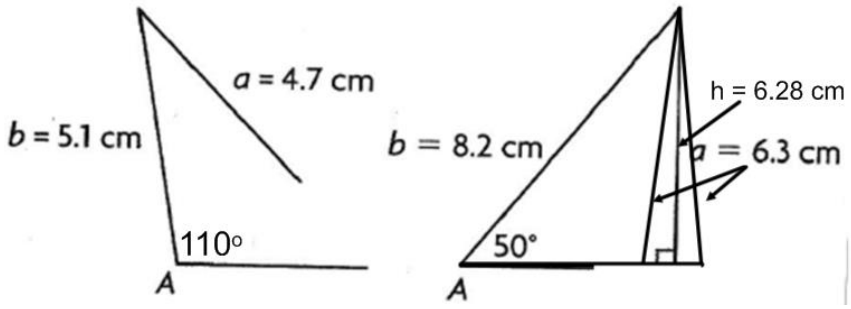
ANSWERS:

1. **a)** *SSA* **b)** *SSA* **c)** not *SSA*

2. **a)** $\angle A$ is acute and $h < a < b$, so two triangles are possible.
b) $\angle D$ is acute and $d > f$, so one triangle is possible.

3. **a)** $h = 8.8$ cm; $\angle R$ is obtuse and $r > s$, so one triangle is possible.
b) $h = 5.4$ cm; $\angle X$ is acute and $x < h$, so no triangle is possible.
c) $h = 27.8$ cm; $\angle A$ is acute and $h < a < b$, so two triangles are possible.

4. **a)** no triangle **b)** two possible triangles



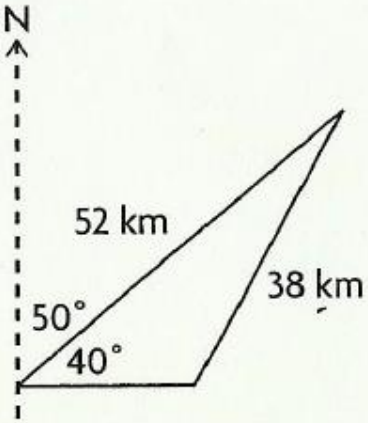
5. In $\triangle XYZ$, $\angle X = 67°$, $x = 3.2$ m, and $y = 3.4$ m.
a) $\doteq 78°$ **b)** $\doteq 102°$; e.g., $\angle X$ is acute, so $\angle Y$ could be obtuse.
c) e.g., $h < x < y$, so there are two possible triangles. In one of these triangles, $\angle Y$ is acute; in the other, $\angle Y$ is obtuse. So, both values for $\angle Y$ correspond to possible triangles.

6. C. **7.** B. **8.** B.

9. $\angle A \doteq 30°$ or $150°$; There are two possible triangles; e.g., $\angle B$ is acute, so $\angle A$ could be either acute or obtuse.

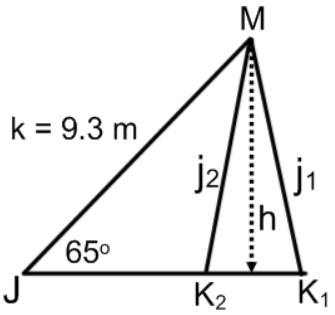
10. $\angle E \doteq 64°$; This is the only possible answer; e,g., $\angle D$ is obtuse, so $\angle E$ must be acute.

11. **a)** e.g.,



b) e.g., $38 > 52 \sin 40°$, so the situation could be ambiguous; however, the fact that the second leg is southwest means that the flight creates an obtuse triangle, removing one possibility.
c) $\doteq 21.8$ km

12. a.



j = 8.7 m

h = 9.3sin65° = 8.4 m

h < j < k, so there are 2 triangles possible.

b.

case 1:	*case* 2:
$\angle K = 75.7^o$	$\angle K = 104.3^o$
$\angle M = 39.3^o$	$\angle M = 10.7^o$
$JK = 6.1\ m$	$JK = 1.8\ m$

c. case 1: area = 25.6 m^2 case 2: area = 7.6 m^2