



Properties of Angles and Triangles

► LEARNING GOALS

You will be able to develop your spatial sense by

- Proving properties of angles formed by intersecting lines
- Proving properties of angles in triangles and other polygons
- Using proven properties to solve geometric problems

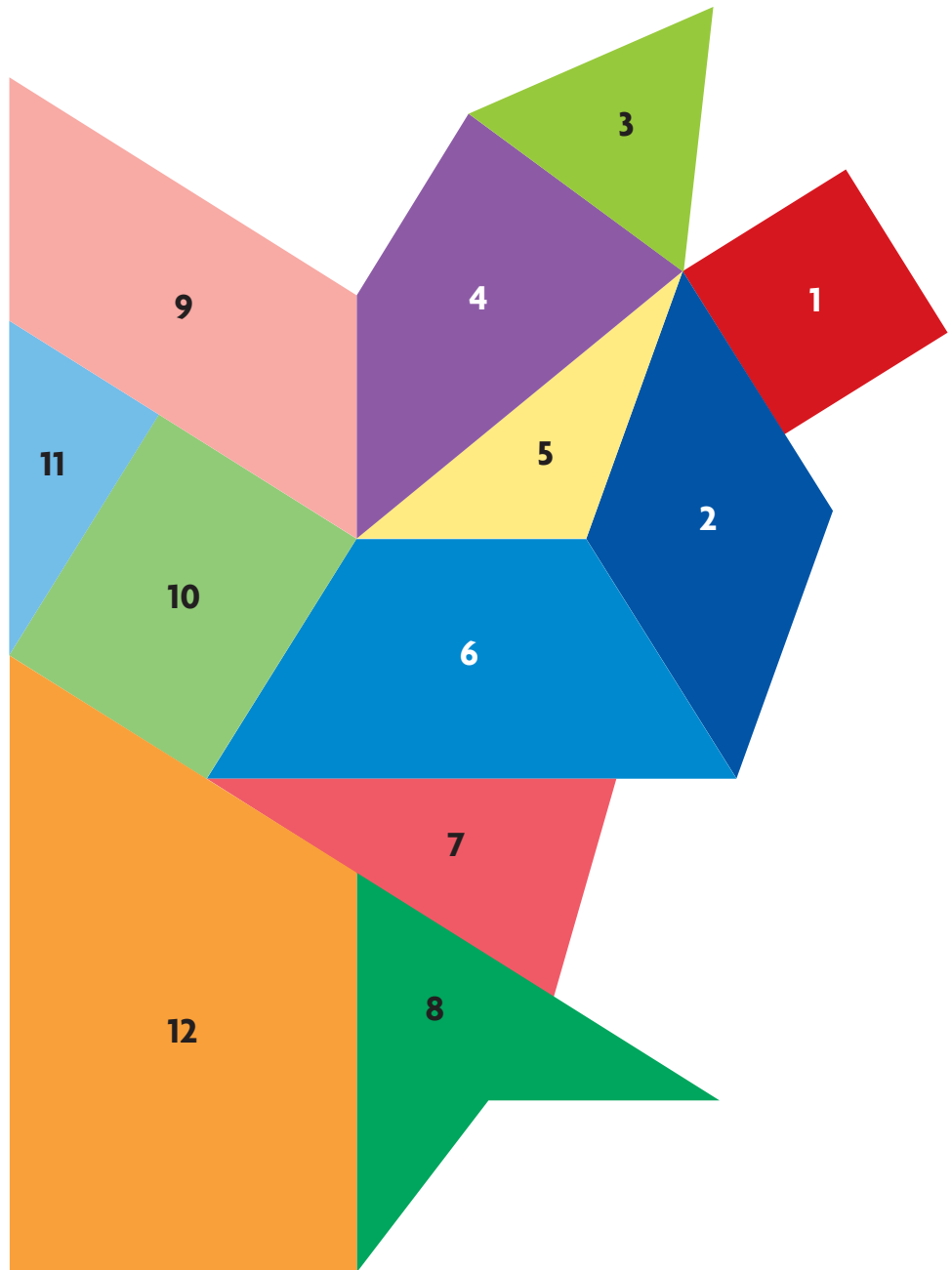
? The Museum of Anthropology at the University of British Columbia houses approximately 6000 archaeological objects from British Columbia's First Nations. Arthur Erickson, a Vancouver-born architect, designed this world-renowned museum. How did he use geometry to enhance his design?

Geometric Art

YOU WILL NEED

- ruler
- protractor
- table for Getting Started

Fawntana used polygons to represent a dog as a mosaic for her art class.



? What rules can you use to sort these polygons?

- A. With a partner, sort the polygons in Fawntana's art.
- B. Compare your sorting with the sortings of other students, and discuss the rules used for each.
- C. Record your sorting in a table like the one below, including the following polygons: quadrilateral, trapezoid, parallelogram, rhombus, rectangle, square, triangle, scalene triangle, isosceles triangle, equilateral triangle, acute triangle, obtuse triangle, and right triangle.

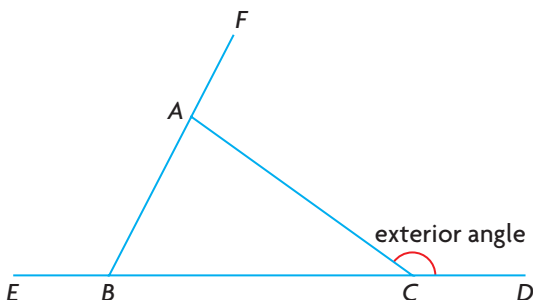
Polygon	Properties	Polygon in Mosaic
quadrilateral	has four sides	1, 2, 4, 6, 8, 9, 10, 12
trapezoid	has one pair of parallel sides	

- D. Create your own mosaic, using at least four different polygons. Classify the polygons you used, and explain your classification.

WHAT DO You Think?

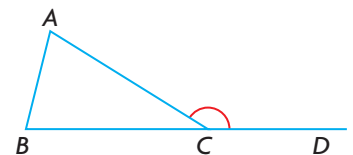
Decide whether you agree or disagree with each statement. Explain your decision.

- There is a specific relationship between parallel lines and the angles formed by these lines and other lines that intersect them.
- The sum of the measures of the interior angles of a triangle is 180° , so the sum of the measures of the **exterior angles** around a triangle is also 180° .



exterior angle of a polygon

The angle that is formed by a side of a polygon and the extension of an adjacent side.



$\angle ACD$ is an exterior angle of $\triangle ABC$.

YOU WILL NEED

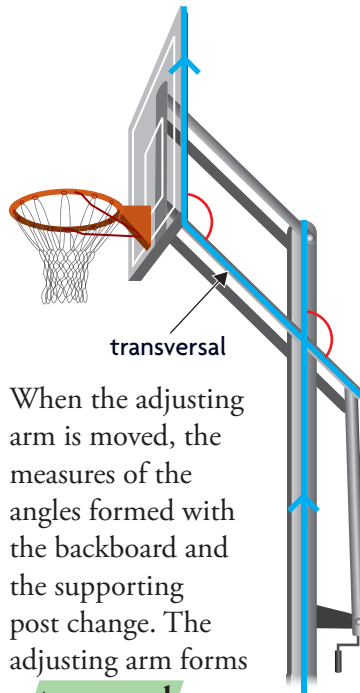
- dynamic geometry software
OR ruler and protractor

GOAL

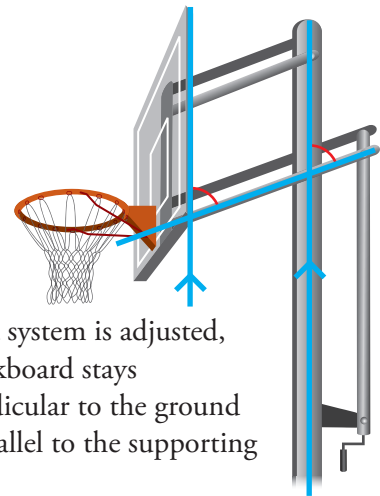
Identify relationships among the measures of angles formed by intersecting lines.

EXPLORE the Math

A sports equipment manufacturer builds portable basketball systems, like those shown here. These systems can be adjusted to different heights.



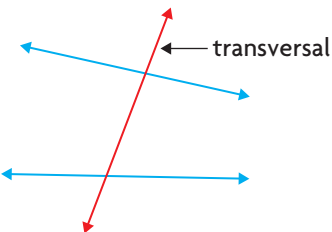
When the adjusting arm is moved, the measures of the angles formed with the backboard and the supporting post change. The adjusting arm forms a **transversal**.



When a system is adjusted, the backboard stays perpendicular to the ground and parallel to the supporting post.

transversal

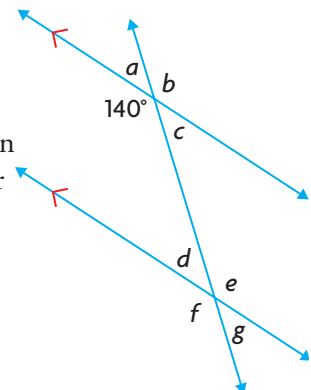
A line that intersects two or more other lines at distinct points.



- ?** When a transversal intersects two parallel lines, how are the angle measures related?

Reflecting

- A.** Use the relationships you observed to predict the measures of as many of the angles a to g in this diagram as you can. Explain each of your predictions.

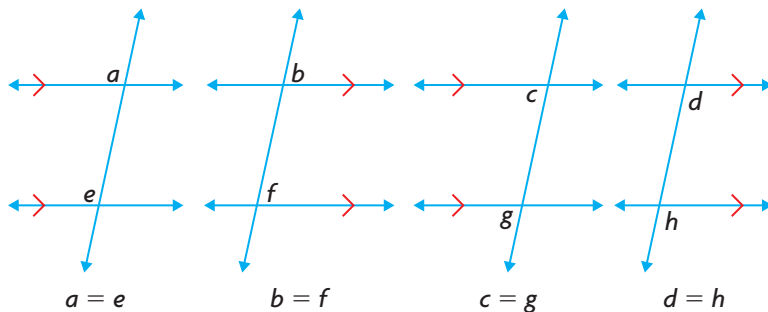


- B. Jonathan made the following conjecture: “When a transversal intersects two parallel lines, the **corresponding angles** are always equal.” Do you agree or disagree? Explain, using examples.
- C. Did you discover any counterexamples for Jonathan’s conjecture? What does this imply?
- D. Sarah says that the **converse** of Jonathan’s conjecture is also true: “When a transversal intersects two lines and creates corresponding angles that are equal, the two lines are parallel.” Do you agree or disagree? Explain.
- E. Do your conjectures about angle measures hold when a transversal intersects a pair of non-parallel lines? Use diagrams to justify your decision.

In Summary

Key Ideas

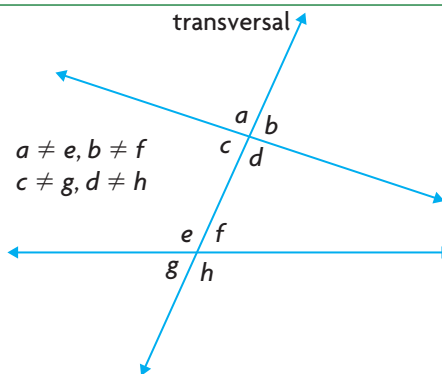
- When a transversal intersects a pair of parallel lines, the corresponding angles that are formed by each parallel line and the transversal are equal.



- When a transversal intersects a pair of lines creating equal corresponding angles, the pair of lines is parallel.

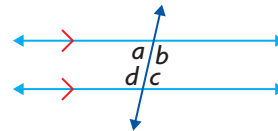
Need to Know

- When a transversal intersects a pair of non-parallel lines, the corresponding angles are not equal.
- There are also other relationships among the measures of the eight angles formed when a transversal intersects two parallel lines.



interior angles

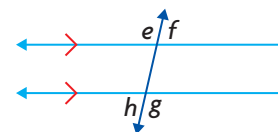
Any angles formed by a transversal and two parallel lines that lie inside the parallel lines.



$a, b, c,$ and d are interior angles.

exterior angles

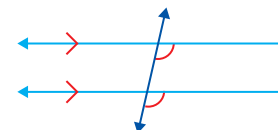
Any angles formed by a transversal and two parallel lines that lie outside the parallel lines.



$e, f, g,$ and h are exterior angles.

corresponding angles

One interior angle and one exterior angle that are non-adjacent and on the same side of a transversal.



converse

A statement that is formed by switching the premise and the conclusion of another statement.

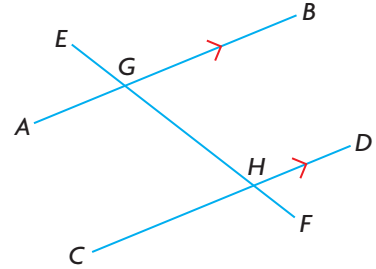


Edmonton's High Level Bridge was designed to carry trains, streetcars, autos, and pedestrians over the North Saskatchewan River. The railway has since been closed, but streetcars still cross, mainly as a tourist attraction.

FURTHER Your Understanding

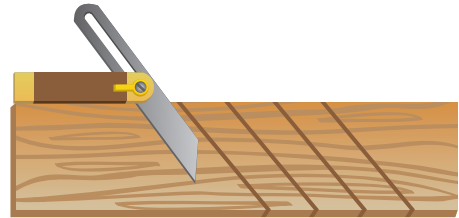
1. a) Identify examples of parallel lines and transversals in this photograph of the High Level Bridge in Edmonton.
b) Can you show that the lines in your examples really are parallel by measuring angles in a tracing of the photograph? Explain.

2. Which pairs of angles are equal in this diagram? Is there a relationship between the measures of the pairs of angles that are not equal?

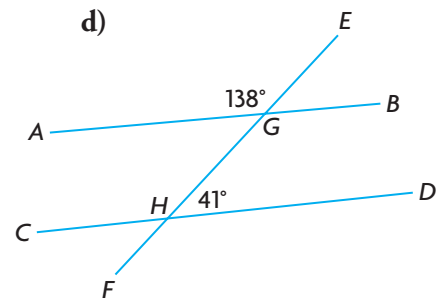
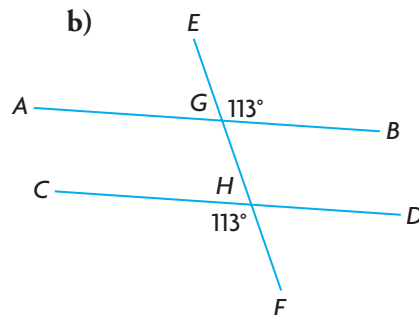
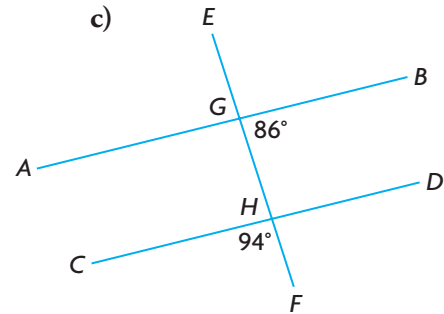
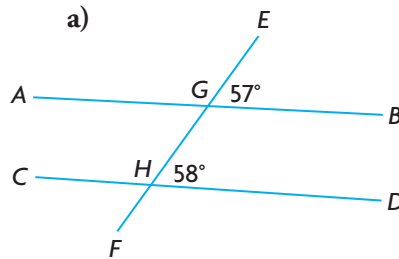


3. Explain how you could construct parallel lines using only a protractor and a ruler.

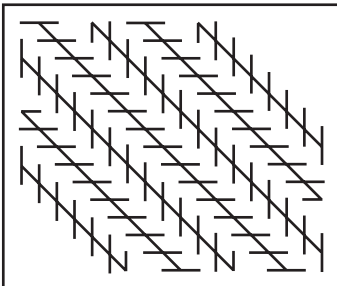
4. An adjustable T-bevel is used to draw parallel lines on wood to indicate where cuts should be made. Explain where the transversal is located in the diagram and how a T-bevel works.



5. In each diagram, is AB parallel to CD ? Explain how you know.



6. Nancy claims that the diagonal lines in the diagram to the left are not parallel. Do you agree or disagree? Justify your decision.



GOAL

Prove properties of angles formed by parallel lines and a transversal, and use these properties to solve problems.

INVESTIGATE the Math

Briony likes to use parallel lines in her art. To ensure that she draws the parallel lines accurately, she uses a straight edge and a compass.

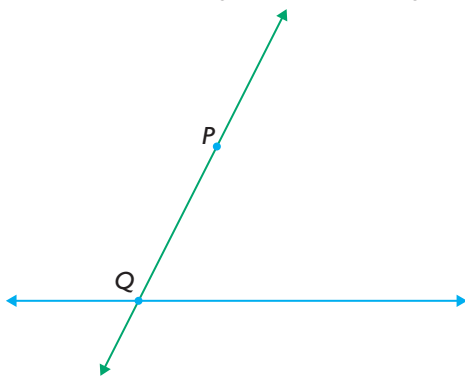
? How can Briony use a straight edge and a compass to ensure that the lines she draws really are parallel?

- A.** Draw the first line. Place a point, labelled P , above the line. P will be a point in a parallel line.

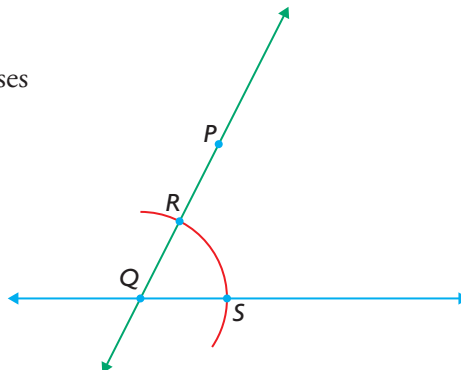
P .



- B.** Draw a line through P , intersecting the first line at Q .



- C.** Using a compass, construct an arc that is centred at Q and passes through both lines. Label the intersection points R and S .

**YOU WILL NEED**

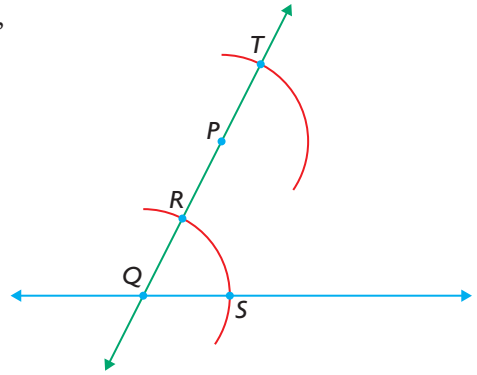
- compass
- protractor
- ruler

**EXPLORE...**

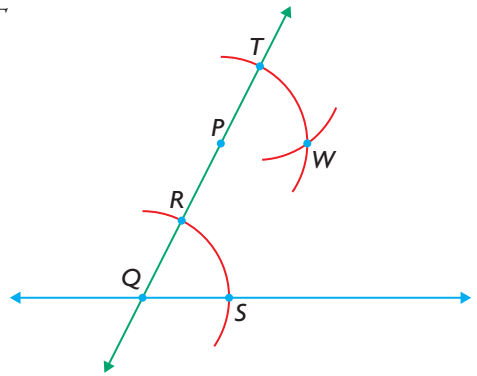
- Parallel bars are used in therapy to help people recover from injuries to their legs or spine. How could the manufacturer ensure that the bars are actually parallel?



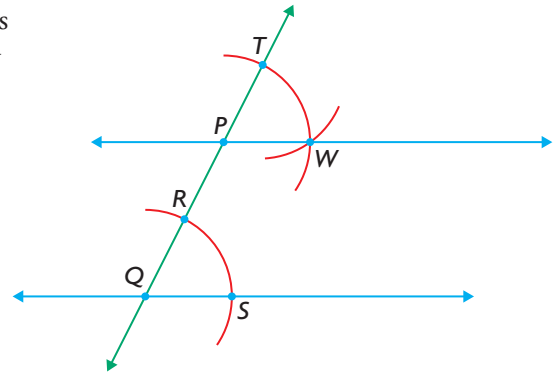
- D. Draw another arc, centred at P , with the same radius as arc RS . Label the intersection point T .



- E. Draw a third arc, with centre T and radius RS , that intersects the arc you drew in step D. Label the point of intersection W .



- F. Draw the line that passes through P and W . Show that $PW \parallel QS$.



Communication **Tip**

If lines PW and QS are parallel, you can represent the relationship using the symbol \parallel :
 $PW \parallel QS$

Reflecting

- G. How is $\angle SQR$ related to $\angle WPT$?
- H. Explain why the compass technique you used ensures that the two lines you drew are parallel.
- I. Are there any other pairs of equal angles in your construction? Explain.

APPLY the Math

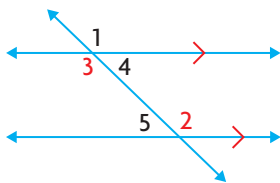
EXAMPLE 1

Reasoning about conjectures involving angles formed by transversals

Make a conjecture that involves the interior angles formed by parallel lines and a transversal. Prove your conjecture.

Tuyet's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the **alternate interior angles** are equal.



I drew two parallel lines and a transversal as shown, and I numbered the angles. I need to show that $\angle 3 = \angle 2$.

Statement	Justification
$\angle 1 = \angle 2$	Corresponding angles
$\angle 1 = \angle 3$	Vertically opposite angles
$\angle 3 = \angle 2$	Transitive property
My conjecture is proved.	

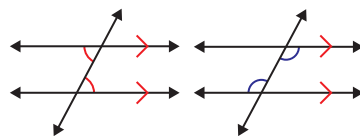
Since I know that the lines are parallel, the corresponding angles are equal.

When two lines intersect, the opposite angles are equal.

$\angle 2$ and $\angle 3$ are both equal to $\angle 1$, so $\angle 2$ and $\angle 3$ are equal to each other.

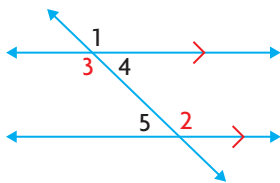
alternate interior angles

Two non-adjacent interior angles on opposite sides of a transversal.



Ali's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the interior angles on the same side of the transversal are supplementary.



$$\angle 1 = \angle 2$$

$$\angle 2 + \angle 5 = 180^\circ$$

I need to show that $\angle 3$ and $\angle 5$ are supplementary.

Since the lines are parallel, the corresponding angles are equal.

These angles form a straight line, so they are supplementary.



$$\angle 1 + \angle 5 = 180^\circ$$

Since $\angle 2 = \angle 1$, I could substitute $\angle 1$ for $\angle 2$ in the equation.

$$\angle 1 = \angle 3$$

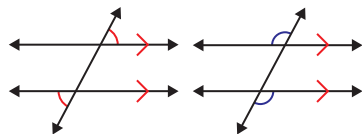
Vertically opposite angles are equal. Since $\angle 1 = \angle 3$, I could substitute $\angle 3$ for $\angle 1$ in the equation.

$$\angle 3 + \angle 5 = 180^\circ$$

My conjecture is proved.

alternate exterior angles

Two exterior angles formed between two lines and a transversal, on opposite sides of the transversal.



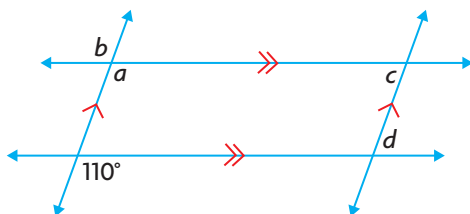
Your Turn

Naveen made the following conjecture: “**Alternate exterior angles** are equal.” Prove Naveen’s conjecture.

EXAMPLE 2

Using reasoning to determine unknown angles

Determine the measures of a , b , c , and d .



Kebbeh's Solution

$$\angle a = 110^\circ$$

The 110° angle and $\angle a$ are corresponding. Since the lines are parallel, the 110° angle and $\angle a$ are equal.

$$\angle a = \angle b$$

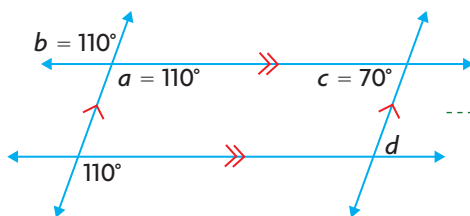
Vertically opposite angles are equal.

$$\angle b = 110^\circ$$

$$\angle c + \angle a = 180^\circ$$

$$\angle c + 110^\circ = 180^\circ$$

$$\angle c = 70^\circ$$



$\angle c$ and $\angle a$ are interior angles on the same side of a transversal. Since the lines are parallel, $\angle c$ and $\angle a$ are supplementary.

I updated the diagram.

$$\angle c = \angle d$$

$$\angle d = 70^\circ$$

$\angle c$ and $\angle d$ are alternate interior angles. Since the lines are parallel, $\angle c$ and $\angle d$ are equal.

The measures of the angles are:

$$\angle a = 110^\circ; \angle b = 110^\circ;$$

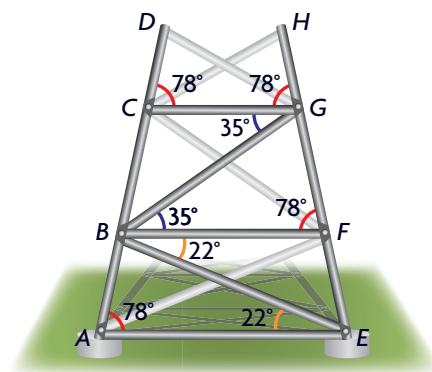
$$\angle c = 70^\circ; \angle d = 70^\circ.$$

Your Turn

- Describe a different strategy you could use to determine the measure of $\angle b$.
- Describe a different strategy you could use to determine the measure of $\angle d$.

EXAMPLE 3 Using angle properties to prove that lines are parallel

One side of a cellphone tower will be built as shown. Use the angle measures to prove that braces CG , BF , and AE are parallel.



Morteza's Solution: Using corresponding angles

$\angle BAE = 78^\circ$ and $\angle DCG = 78^\circ$	Given
$AE \parallel CG$	When corresponding angles are equal, the lines are parallel.
$\angle CGH = 78^\circ$ and $\angle BFG = 78^\circ$	Given
$CG \parallel BF$	When corresponding angles are equal, the lines are parallel.
$AE \parallel CG$ and $CG \parallel BF$	Since AE and BF are both parallel to CG , all three lines are parallel to each other.

The three braces are parallel.

Jennifer's Solution: Using alternate interior angles

Statement	Justification	
$\angle CGB = 35^\circ$ and $\angle GBF = 35^\circ$	Given	
$CG \parallel BF$	Alternate interior angles	When alternate interior angles are equal, the lines are parallel.
$\angle FBE = 22^\circ$ and $\angle BEA = 22^\circ$	Given	
$BF \parallel AE$	Alternate interior angles	When alternate interior angles are equal, the lines are parallel.
$CG \parallel BF$ and $BF \parallel AE$	Transitive property	Since CG and AE are both parallel to BF , they must also be parallel to each other.
The three braces are parallel.		

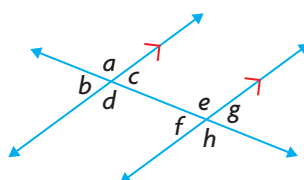
Your Turn

Use a different strategy to prove that CG , BF , and AE are parallel.

In Summary

Key Idea

- When a transversal intersects two parallel lines,
 - the corresponding angles are equal.
 - the alternate interior angles are equal.
 - the alternate exterior angles are equal.
 - the interior angles on the same side of the transversal are supplementary.



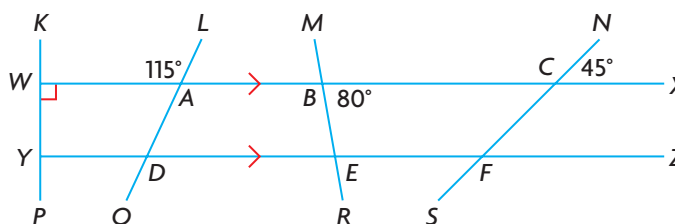
- $a = e, b = f$
 $c = g, d = h$
- $c = f, d = e$
- $a = h, b = g$
- $c + e = 180^\circ$
 $d + f = 180^\circ$

Need to Know

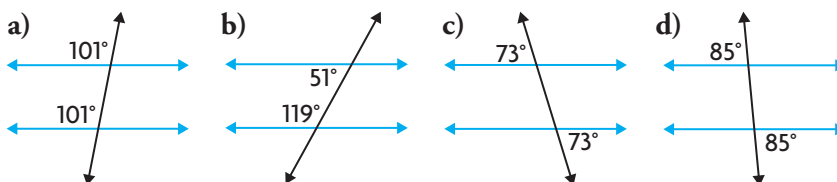
- If a transversal intersects two lines such that
 - the corresponding angles are equal, or
 - the alternate interior angles are equal, or
 - the alternate exterior angles are equal, or
 - the interior angles on the same side of the transversal are supplementary,
 then the lines are parallel.

CHECK Your Understanding

- Determine the measures of $\angle WYD$, $\angle YDA$, $\angle DEB$, and $\angle EFS$. Give your reasoning for each measure.



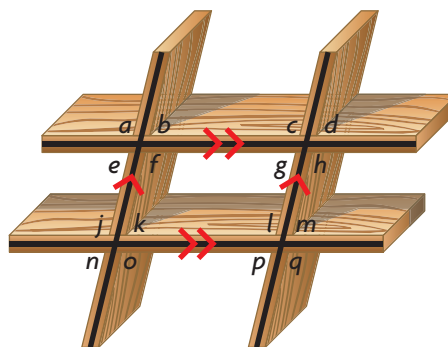
- For each diagram, decide if the given angle measures prove that the blue lines are parallel. Justify your decisions.



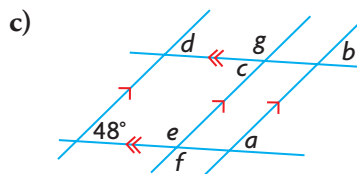
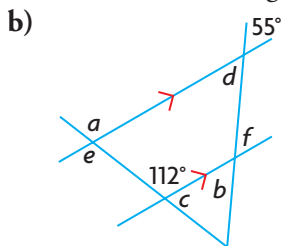
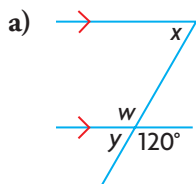
PRACTISING

3. A shelving unit is built with two pairs of parallel planks. Explain why each of the following statements is true.

- a) $\angle k = \angle p$ e) $\angle b = \angle m$
 b) $\angle a = \angle j$ f) $\angle e = \angle p$
 c) $\angle j = \angle q$ g) $\angle n = \angle d$
 d) $\angle g = \angle d$ h) $\angle f + \angle k = 180^\circ$



4. Determine the measures of the indicated angles.



5. Construct an isosceles trapezoid. Explain your method of construction.

6. a) Construct parallelogram $SHOE$, where $\angle S = 50^\circ$.
 b) Show that the opposite angles of parallelogram $SHOE$ are equal.
7. a) Identify pairs of parallel lines and transversals in the embroidery pattern.
 b) How could a pattern maker use the properties of the angles created by parallel lines and a transversal to draw an embroidery pattern accurately?
8. a) Joshua made the following conjecture: “If $AB \perp BC$ and $BC \perp CD$, then $AB \perp CD$.” Identify the error in his reasoning.

Joshua's Proof

Statement	Justification
$AB \perp BC$	Given
$BC \perp CD$	Given
$AB \perp CD$	Transitive property

- b) Make a correct conjecture about perpendicular lines.



Embroidery has a rich history in the Ukrainian culture. In one style of embroidery, called *nabiruvannia*, the stitches are made parallel to the horizontal threads of the fabric.



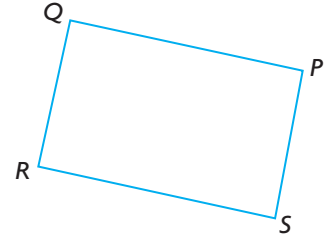
The Bank of China tower has a distinctive 3-D shape. The base of the lower part of the building is a quadrilateral. The base of the top is a triangle, making it stable and wind-resistant.

9. The Bank of China tower in Hong Kong was the tallest building in Asia at the time of its completion in 1990. Explain how someone in Hong Kong could use angle measures to determine if the diagonal trusses are parallel.

10. Jason wrote the following proof. Identify his errors, and correct his proof.

Given: $QP \perp QR$
 $QR \perp RS$
 $QR \parallel PS$

Prove: $QPSR$ is a parallelogram.



Jason's Proof

Statement	Justification
$\angle PQR = 90^\circ$ and $\angle QRS = 90^\circ$	Lines that are perpendicular meet at right angles.
$QP \parallel RS$	Since the interior angles on the same side of a transversal are equal, QP and RS are parallel.
$QR \parallel PS$ $QPSR$ is a parallelogram	Given $QPSR$ has two pairs of parallel sides.

11. The roof of St. Ann's Academy in Victoria, British Columbia, has dormer windows as shown. Explain how knowledge of parallel lines and transversals helped the builders ensure that the frames for the windows are parallel.



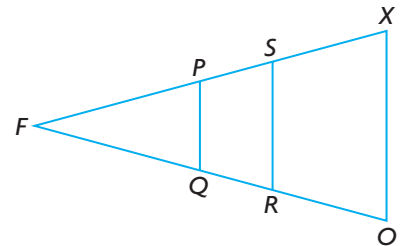
12. Given: $\triangle FOX$ is isosceles.

$$\angle FOX = \angle FRS$$

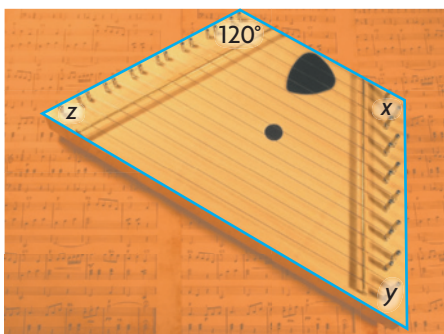
$$\angle FXO = \angle FPQ$$

Prove: $PQ \parallel SR$ and $SR \parallel XO$

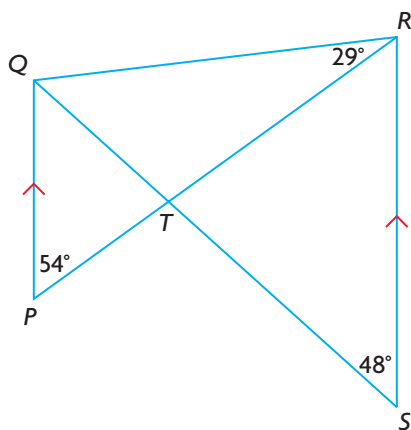
13. a) Draw a triangle. Construct a line segment that joins two sides of your triangle and is parallel to the third side.
 b) Prove that the two triangles in your construction are similar.



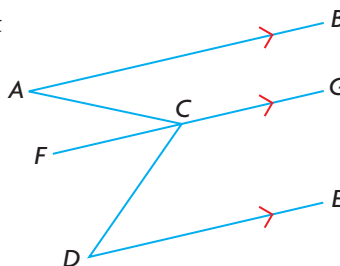
14. The top surface of this lap harp is an isosceles trapezoid.
- Determine the measures of the unknown angles.
 - Make a conjecture about the angles in an isosceles trapezoid.



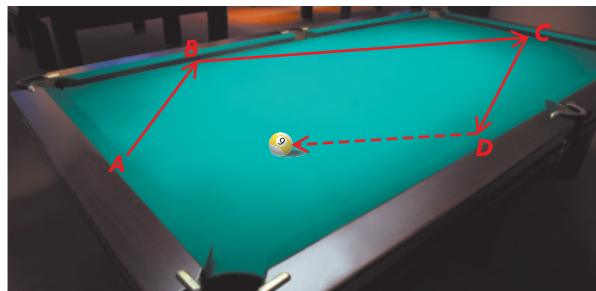
15. Determine the measures of all the unknown angles in this diagram, given $PQ \parallel RS$.



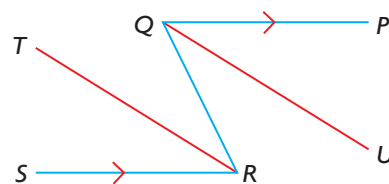
16. Given $AB \parallel DE$ and $DE \parallel FG$, show that $\angle ACD = \angle BAC + \angle CDE$.



17. When a ball is shot into the side or end of a pool table, it will rebound off the side or end at the same angle that it hit (assuming that there is no spin on the ball).
- Predict how the straight paths of the ball will compare with each other.
 - Draw a scale diagram of the top of a pool table that measures 4 ft by 8 ft. Construct the trajectory of a ball that is hit from point A on one end toward point B on a side, then C , D , and so on.
 - How does path AB compare with path CD ? How does path BC compare with path DE ? Was your prediction correct?
 - Will this pattern continue? Explain.

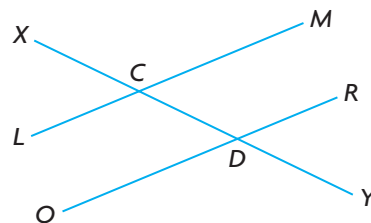


18. Given: $QP \parallel SR$
 RT bisects $\angle QRS$.
 QU bisects $\angle PQR$.
 Prove: $QU \parallel RT$



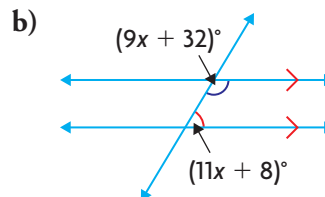
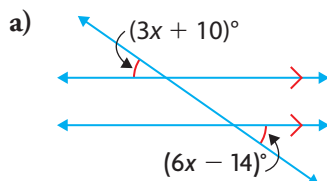
Closing

19. a) Ashley wants to prove that $LM \parallel QR$. To do this, she claims that she must show all of the following statements to be true:
 i) $\angle LCD = \angle CDR$
 ii) $\angle XCM = \angle CDR$
 iii) $\angle MCD + \angle CDR = 180^\circ$
 Do you agree or disagree? Explain.
 b) Can Ashley show that the lines are parallel in other ways? If so, list these ways.



Extending

20. Solve for x .



21. The window surface of the large pyramid at Edmonton City Hall is composed of congruent rhombuses.
- a) Describe how you could determine the angle at the peak of the pyramid using a single measurement without climbing the pyramid.
- b) Prove that your strategy is valid.



The Edmonton City Hall pyramids are a city landmark.

Applying Problem-Solving Strategies

Checkerboard Quadrilaterals

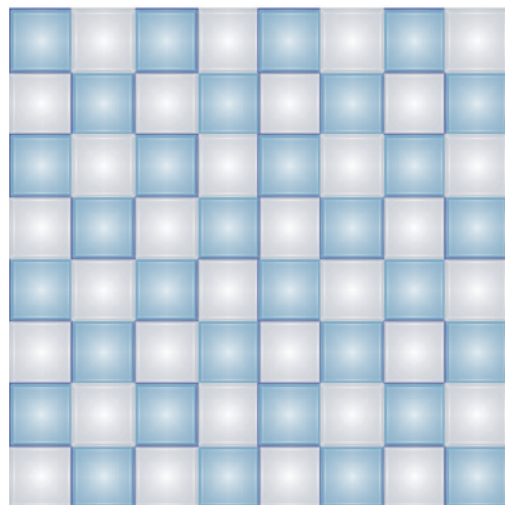
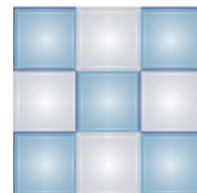
One strategy for solving a puzzle is to use inductive reasoning. Solve similar but simpler puzzles first, then look for patterns in your solutions that may help you solve the original, more difficult puzzle.

The Puzzle

How many quadrilaterals can you count on an 8-by-8 checkerboard?

The Strategy

- A. How many squares or rectangles can you count on a 1-by-1 checkerboard?
- B. Draw a 2-by-2 checkerboard. Count the quadrilaterals.
- C. Draw a 3-by-3 checkerboard, and count the quadrilaterals.
- D. Develop a strategy you could use to determine the number of quadrilaterals on any checkerboard. Test your strategy on a 4-by-4 checkerboard.
- E. Was your strategy effective? Modify your strategy if necessary.
- F. Determine the number of quadrilaterals on an 8-by-8 checkerboard. Describe your strategy.
- G. Compare your results and strategy with the results and strategies of your classmates. Did all the strategies result in the same solution? How many different strategies were used?
- H. Which strategy do you like the best? Explain.



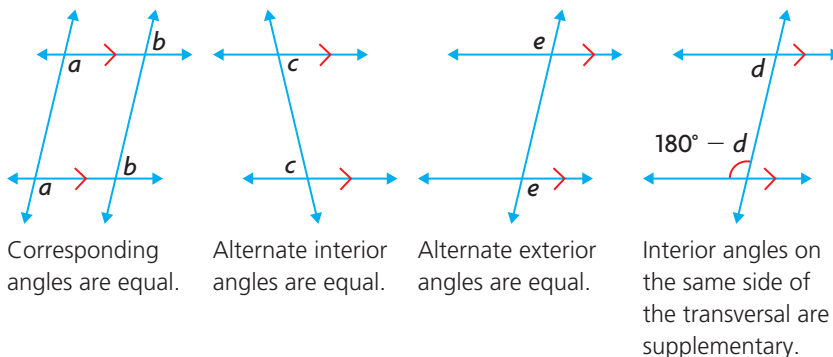
FREQUENTLY ASKED Questions

Study Aid

- See Lesson 2.1 and Lesson 2.2, Example 1.
- Try Mid-Chapter Review Questions 1 to 5.

Q: What are the relationships among the angles formed when a transversal intersects two parallel lines?

A: When two lines are parallel, the following angle relationships hold:



Study Aid

- See Lesson 2.2, Example 2.
- Try Mid-Chapter Review Question 6.

Q: How can you prove that a conjecture involving parallel lines is valid?

A: Draw a diagram that shows parallel lines and a transversal. Label your diagram with any information you know about the lines and angles. State what you know and what you are trying to prove. Make a plan. Use other conjectures that have already been proven to complete each step of your proof.

Study Aid

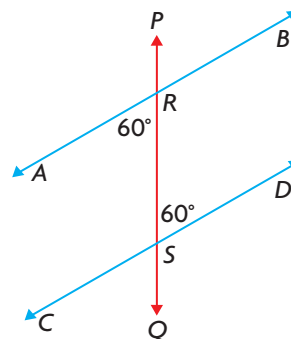
- See Lesson 2.2, Example 3.
- Try Mid-Chapter Review Questions 6 and 8.

Q: How can you use angles to prove that two lines are parallel?

A: Draw a transversal that intersects the two lines, if the diagram does not include a transversal. Then measure, or determine the measure of, a pair of angles formed by the transversal and the two lines. If corresponding angles, alternate interior angles, or alternate exterior angles are equal, or if interior angles on the same side of the transversal are supplementary, the lines are parallel.

For example, determine if AB is parallel to CD . First draw transversal PQ .

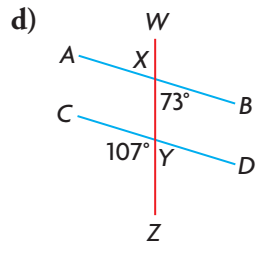
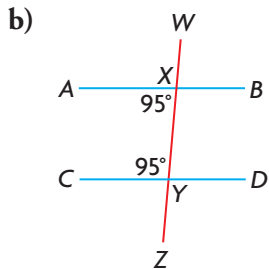
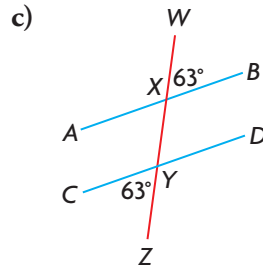
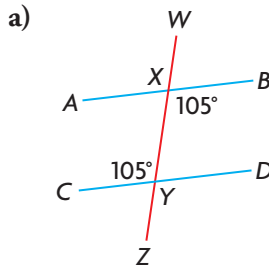
Measure alternate interior angles $\angle ARQ$ and $\angle PSD$. If these angles are equal, then $AB \parallel CD$.



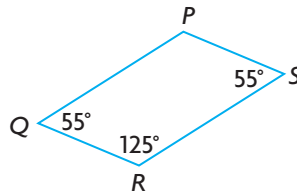
PRACTISING

Lesson 2.1

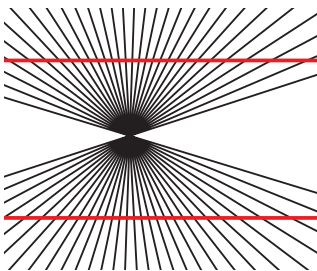
1. In each diagram, determine whether $AB \parallel CD$. Explain how you know.



2. Classify quadrilateral $PQRS$. Explain how you know.



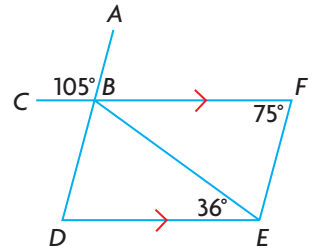
3. Are the red lines in the artwork parallel? Explain.



Lesson 2.2

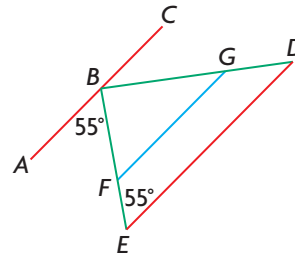
4. Draw a parallelogram by constructing two sets of parallel lines. Explain your method.

5. a) Determine the measures of all the unknown angles in the diagram.
b) Is BD parallel to EF ? Explain how you know.



6. Given: $\triangle BFG \sim \triangle BED$

Prove: a) $AC \parallel ED$
b) $FG \parallel ED$
c) $AC \parallel FG$



7. Explain how knowledge of parallel lines and transversals could be used to determine where to paint the lines for these parking spots.



8. The Franco-Yukonnais flag is shown. Are the long sides of the white shapes parallel to the long sides of the yellow shape? Explain how you know.



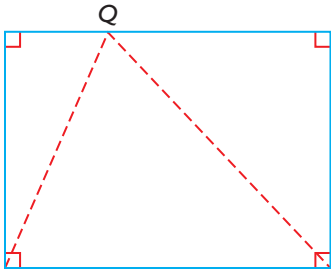
YOU WILL NEED

- dynamic geometry software
OR compass, protractor, and ruler
- scissors

EXPLORE...

On a rectangular piece of paper, draw lines from two vertices to a point on the opposite side. Cut along the lines to create two right triangles and an acute triangle.

- What do you notice about the three triangles?
- Can you use angle relationships to show that the sum of the measures of the angles in any acute triangle formed this way is 180° ?

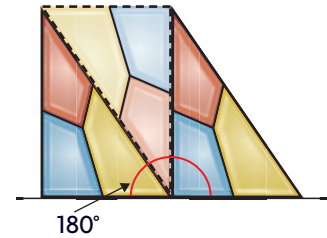


GOAL

Prove properties of angles in triangles, and use these properties to solve problems.

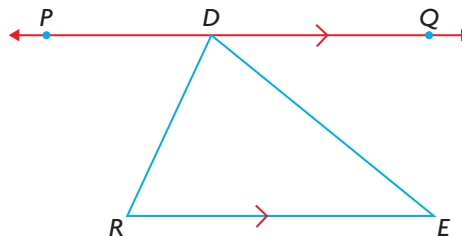
INVESTIGATE the Math

Diko placed three congruent triangular tiles so that a different angle from each triangle met at the same point. She noticed the angles seemed to form a straight line.



- ?** Can you prove that the sum of the measures of the interior angles of any triangle is 180° ?

- A. Draw an acute triangle, $\triangle RED$. Construct line PQ through vertex D , parallel to RE .



- B. Identify pairs of equal angles in your diagram. Explain how you know that the measures of the angles in each pair are equal.
- C. What is the sum of the measures of $\angle PDR$, $\angle RDE$, and $\angle QDE$? Explain how you know.
- D. Explain why:

$$\angle DRE + \angle RDE + \angle RED = 180^\circ$$
- E. In part A, does it matter which vertex you drew the parallel line through? Explain, using examples.
- F. Repeat parts A to E, first for an obtuse triangle and then for a right triangle. Are your results the same as they were for the acute triangle?

Reflecting

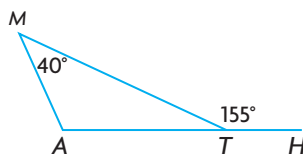
- G. Why is Diko's approach not considered to be a proof?
- H. Are your results sufficient to prove that the sum of the measures of the angles in any triangle is 180° ? Explain.

APPLY the Math

EXAMPLE 1

Using angle sums to determine angle measures

In the diagram, $\angle MTH$ is an **exterior angle** of $\triangle MAT$. Determine the measures of the unknown angles in $\triangle MAT$.



Serge's Solution

$$\angle MTA + \angle MTH = 180^\circ$$

$$\angle MTA + (155^\circ) = 180^\circ$$

$$\angle MTA = 25^\circ$$

$\angle MTA$ and $\angle MTH$ are supplementary since they form a straight line.

$$\angle MAT + \angle AMT + \angle MTA = 180^\circ$$

$$\angle MAT + (40^\circ) + (25^\circ) = 180^\circ$$

$$\angle MAT = 115^\circ$$

The sum of the measures of the interior angles of any triangle is 180° .

The measures of the unknown angles are:

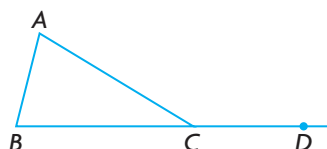
$$\angle MTA = 25^\circ; \angle MAT = 115^\circ.$$

Your Turn

If you are given one interior angle and one exterior angle of a triangle, can you always determine the other interior angles of the triangle? Explain, using diagrams.

non-adjacent interior angles

The two angles of a triangle that do not have the same vertex as an exterior angle.



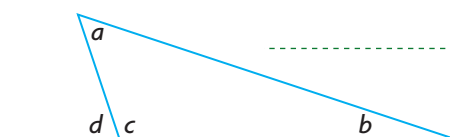
$\angle A$ and $\angle B$ are non-adjacent interior angles to exterior $\angle ACD$.

EXAMPLE 2

Using reasoning to determine the relationship between the exterior and interior angles of a triangle

Determine the relationship between an exterior angle of a triangle and its **non-adjacent interior angles**.

Joanna's Solution



I drew a diagram of a triangle with one exterior angle. I labelled the angle measures a , b , c , and d .

$$\begin{aligned}\angle d + \angle c &= 180^\circ \\ \angle d &= 180^\circ - \angle c\end{aligned}$$

$\angle d$ and $\angle c$ are supplementary. I rearranged these angles to isolate $\angle d$.

$$\begin{aligned}\angle a + \angle b + \angle c &= 180^\circ \\ \angle a + \angle b &= 180^\circ - \angle c\end{aligned}$$

The sum of the measures of the angles in any triangle is 180° .

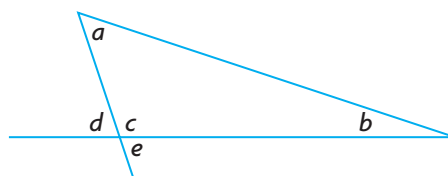
$$\angle d = \angle a + \angle b$$

Since $\angle d$ and $(\angle a + \angle b)$ are both equal to $180^\circ - \angle c$, by the transitive property, they must be equal to each other.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles.

Your Turn

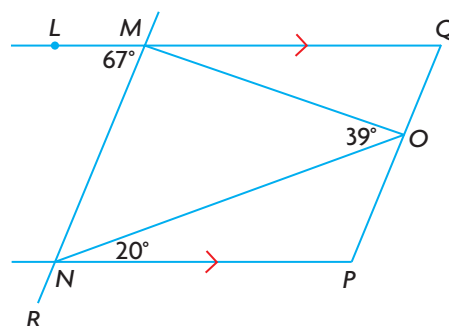
Prove: $\angle e = \angle a + \angle b$



EXAMPLE 3

Using reasoning to solve problems

Determine the measures of $\angle NMO$, $\angle MNO$, and $\angle QMO$.



Tyler's Solution

MN is a transversal of parallel lines LQ and NP .

MN intersects parallel lines LQ and NP .

$$\begin{aligned}\angle MNO + 20^\circ &= 67^\circ \\ \angle MNO &= 47^\circ\end{aligned}$$

Since $\angle LMN$ and $\angle MNP$ are alternate interior angles between parallel lines, they are equal.

$$\begin{aligned}\angle NMO + \angle MNO + 39^\circ &= 180^\circ \\ \angle NMO + (47^\circ) + 39^\circ &= 180^\circ \\ \angle NMO + 86^\circ &= 180^\circ \\ \angle NMO &= 94^\circ\end{aligned}$$

The measures of the angles in a triangle add to 180° .

$$\begin{aligned}\angle NMO + \angle QMO + 67^\circ &= 180^\circ \\ (94^\circ) + \angle QMO + 67^\circ &= 180^\circ \\ 161^\circ + \angle QMO &= 180^\circ \\ \angle QMO &= 19^\circ\end{aligned}$$

$\angle LMN$, $\angle NMO$, and $\angle QMO$ form a straight line, so their measures must add to 180° .

The measures of the angles are:

$$\angle MNO = 47^\circ; \angle NMO = 94^\circ; \angle QMO = 19^\circ.$$

Dominique's Solution

$$\begin{aligned}\angle NMO + \angle MNO + 39^\circ &= 180^\circ \\ \angle NMO + \angle MNO &= 141^\circ\end{aligned}$$

The sum of the measures of the angles in a triangle is 180° .

$$\begin{aligned}(\angle NMO + \angle QMO) + (\angle MNO + 20^\circ) &= 180^\circ \\ \angle NMO + \angle MNO + \angle QMO &= 160^\circ\end{aligned}$$

The angles that are formed by $(\angle NMO + \angle QMO)$ and $(\angle MNO + 20^\circ)$ are interior angles on the same side of transversal MN . Since $LQ \parallel NP$, these angles are supplementary.

$$\begin{aligned}(141^\circ) + \angle QMO &= 160^\circ \\ \angle QMO &= 19^\circ\end{aligned}$$

I substituted the value of $\angle NMO + \angle MNO$ into the equation.

$$\begin{aligned}\angle NMO + \angle QMO + 67^\circ &= 180^\circ \\ \angle NMO + (19^\circ) + 67^\circ &= 180^\circ \\ \angle NMO &= 94^\circ\end{aligned}$$

$\angle LMN$, $\angle NMO$, and $\angle QMO$ form a straight line, so the sum of their measures is 180° .

$$\begin{aligned}\angle NMO + \angle MNO &= 141^\circ \\ (94^\circ) + \angle MNO &= 141^\circ \\ \angle MNO &= 47^\circ\end{aligned}$$

The measures of the angles are:

$$\angle QMO = 19^\circ; \angle NMO = 94^\circ; \angle MNO = 47^\circ.$$

Your Turn

In the diagram for Example 3, $QP \parallel MR$. Determine the measures of $\angle MQO$, $\angle MOQ$, $\angle NOP$, $\angle OPN$, and $\angle RNP$.

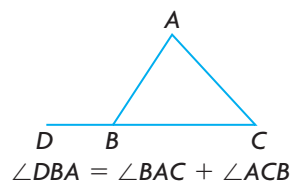
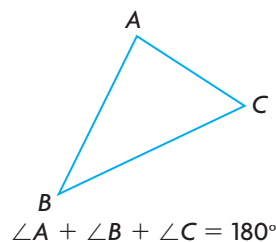
In Summary

Key Idea

- You can prove properties of angles in triangles using other properties that have already been proven.

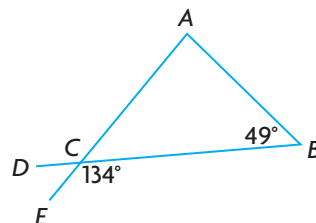
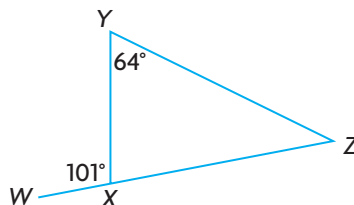
Need to Know

- In any triangle, the sum of the measures of the interior angles is proven to be 180° .
- The measure of any exterior angle of a triangle is proven to be equal to the sum of the measures of the two non-adjacent interior angles.



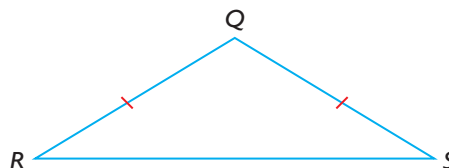
CHECK Your Understanding

- Harrison drew a triangle and then measured the three interior angles. When he added the measures of these angles, the sum was 180° . Does this prove that the sum of the measures of the angles in any triangle is 180° ? Explain.
- Marcel says that it is possible to draw a triangle with two right angles. Do you agree? Explain why or why not.
- Determine the following unknown angles.
 - $\angle YXZ$, $\angle Z$
 - $\angle A$, $\angle DCE$

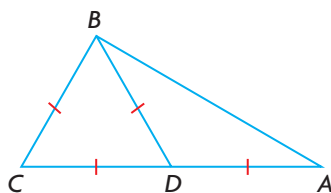


PRACTISING

- If $\angle Q$ is known, write an expression for the measure of one of the other two angles.

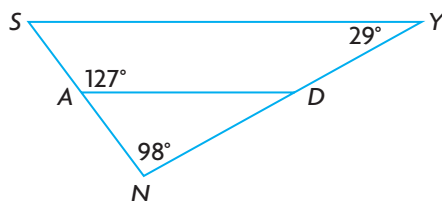


5. Prove: $\angle A = 30^\circ$



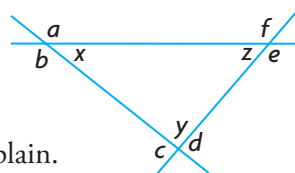
6. Determine the measures of the exterior angles of an equilateral triangle.

7. Prove: $SY \parallel AD$

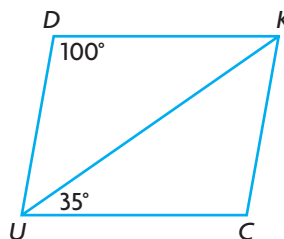


8. Each vertex of a triangle has two exterior angles, as shown.

- Make a conjecture about the sum of the measures of $\angle a$, $\angle c$, and $\angle e$.
- Does your conjecture also apply to the sum of the measures of $\angle b$, $\angle d$, and $\angle f$? Explain.
- Prove or disprove your conjecture.



9. *DUCK* is a parallelogram. Benji determined the measures of the unknown angles in *DUCK*. Paula says he has made an error.



Benji's Solution

Statement

$$\angle DKU = \angle KUC$$

$$\angle DKU = 35^\circ$$

$$\angle UDK = \angle DUC$$

$$\angle DUK + \angle KUC = 100^\circ$$

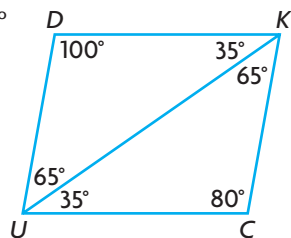
$$\angle DUK = 65^\circ$$

$$\angle UKC = 65^\circ$$

$$\angle UCK = 180^\circ - (\angle KUC + \angle UKC)$$

$$\angle UCK = 180^\circ - (35^\circ + 65^\circ)$$

$$\angle UCK = 80^\circ$$



Justification

$\angle DKU$ and $\angle KUC$ are alternate interior angles.

$\angle UDK$ and $\angle DUC$ are corresponding angles.

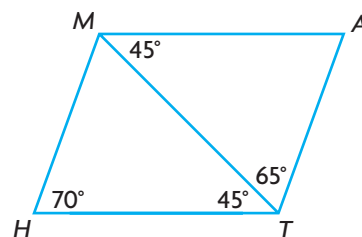
$\angle DUK$ and $\angle UKC$ are alternate interior angles.

The sum of the measures of the angles in a triangle is 180° .

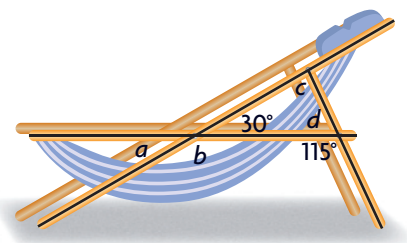
I redrew the diagram, including the angle measures I determined.

- Explain how you know that Benji made an error.
- Correct Benji's solution.

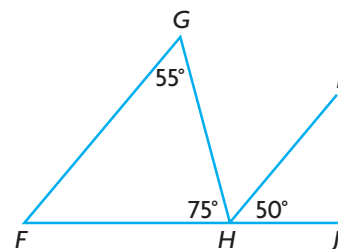
10. Prove that quadrilateral $MATH$ is a parallelogram.



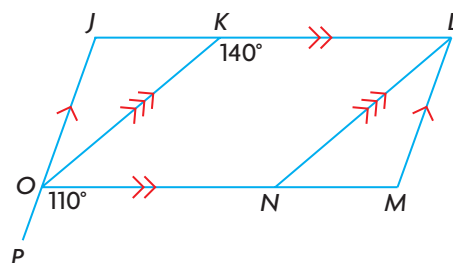
11. A manufacturer is designing a reclining lawn chair, as shown. Determine the measures of $\angle a$, $\angle b$, $\angle c$, and $\angle d$.



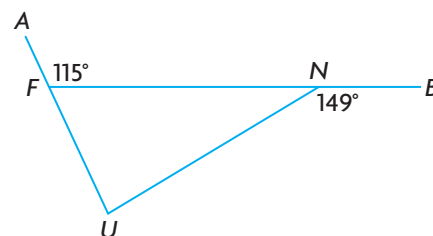
12. a) Tim claims that FG is not parallel to HI because $\angle FGH \neq \angle IHJ$. Do you agree or disagree? Justify your decision.
b) How else could you justify your decision? Explain.



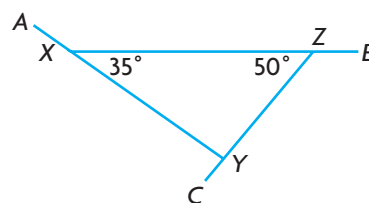
13. Use the given information to determine the measures of $\angle J$, $\angle JKO$, $\angle JOK$, $\angle KLM$, $\angle KLN$, $\angle M$, $\angle LNO$, $\angle LNM$, $\angle MLN$, $\angle NOK$, and $\angle JON$.



14. Determine the measures of the interior angles of $\triangle FUN$.

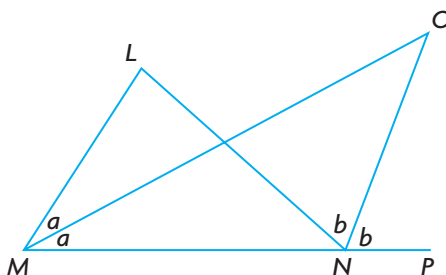


15. a) Determine the measures of $\angle AXZ$, $\angle XYZ$, and $\angle EYZ$.
b) Determine the sum of these three exterior angles.



16. MO and NO are angle bisectors.

Prove: $\angle L = 2\angle O$



Closing

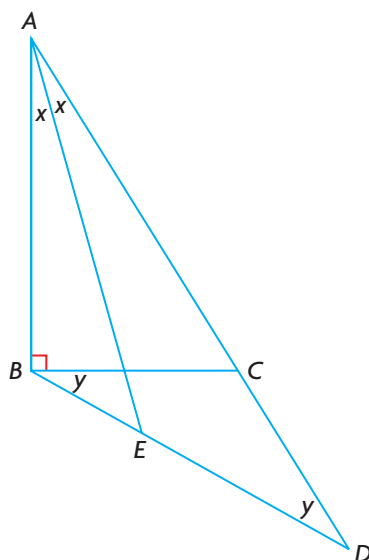
17. Explain how drawing a line that is parallel to one side of any triangle can help you prove that the sum of the angles in the triangle is 180° .

Extending

18. Given: AE bisects $\angle BAC$.

$\triangle BCD$ is isosceles.

Prove: $\angle AEB = 45^\circ$



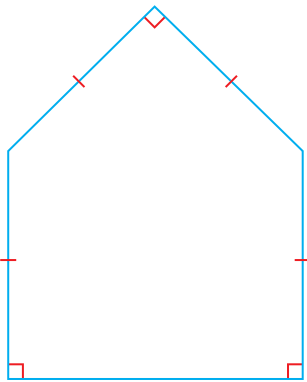
19. $\triangle LMN$ is an isosceles triangle in which $LM = LN$. ML is extended to point D , forming an exterior angle, $\angle DLN$. If $LR \parallel MN$, where N and R are on the same side of MD , prove that $\angle DLR = \angle RLN$.

YOU WILL NEED

- dynamic geometry software
OR protractor and ruler

EXPLORE...

- A pentagon has three right angles and four sides of equal length, as shown. What is the sum of the measures of the angles in the pentagon?



GOAL

Determine properties of angles in polygons, and use these properties to solve problems.

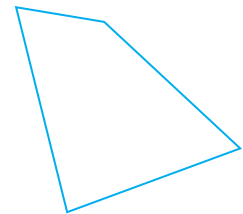
INVESTIGATE the Math

In Lesson 2.3, you proved properties involving the interior and exterior angles of triangles. You can use these properties to develop general relationships involving the interior and exterior angles of polygons.

- ❓ How is the number of sides in a polygon related to the sum of its interior angles and the sum of its exterior angles?

Part 1 Interior Angles

- A. Giuseppe says that he can determine the sum of the measures of the interior angles of this quadrilateral by including the diagonals in the diagram. Is he correct? Explain.
- B. Determine the sum of the measures of the interior angles of any quadrilateral.
- C. Draw the polygons listed in the table below. Create triangles to help you determine the sum of the measures of their interior angles. Record your results in a table like the one below.

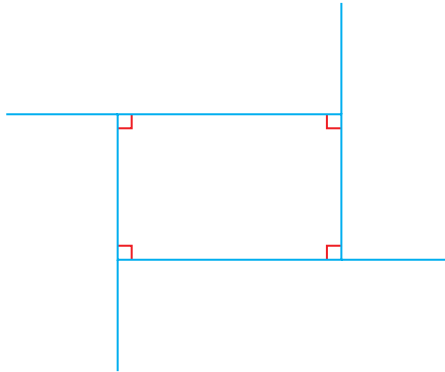


Polygon	Number of Sides	Number of Triangles	Sum of Angle Measures
triangle	3	1	180°
quadrilateral	4		
pentagon	5		
hexagon	6		
heptagon	7		
octagon	8		

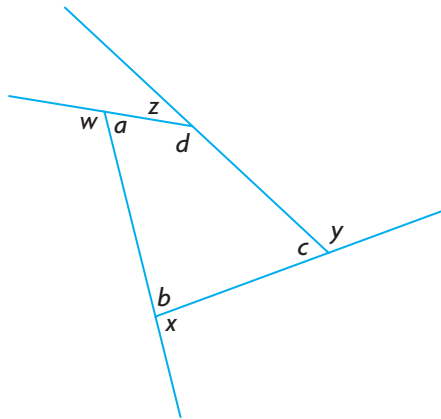
- D. Make a conjecture about the relationship between the sum of the measures of the interior angles of a polygon, S , and the number of sides of the polygon, n .
- E. Use your conjecture to predict the sum of the measures of the interior angles of a dodecagon (12 sides). Verify your prediction using triangles.

Part 2 Exterior Angles

- F. Draw a rectangle. Extend each side of the rectangle so that the rectangle has one exterior angle for each interior angle. Determine the sum of the measures of the exterior angles.

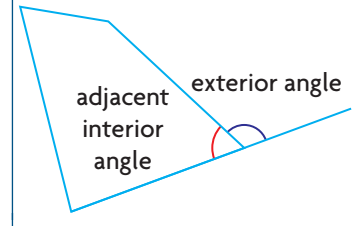


- G. What do you notice about the sum of the measures of each exterior angle of your rectangle and its adjacent interior angle? Would this relationship also hold for the exterior and interior angles of the irregular quadrilateral shown? Explain.



Communication **Tip**

When a side of a polygon is extended, two angles are created. The angle that is considered to be the exterior angle is adjacent to the interior angle at the vertex.



- H. Make a conjecture about the sum of the measures of the exterior angles of any quadrilateral. Test your conjecture.
- I. Draw a pentagon. Extend each side of the pentagon so that the pentagon has one exterior angle for each interior angle. Based on your diagram, revise your conjecture to include pentagons. Test your revised conjecture.
- J. Do you think your revised conjecture will hold for polygons that have more than five sides? Explain and verify by testing.

Reflecting

- K. Compare your results for the sums of the measures of the interior angles of polygons with your classmates' results. Do you think your conjecture from part D will be true for any polygon? Explain.
- L. Compare your results for the sums of the measures of the exterior angles of polygons with your classmates' results. Do you think your conjecture from part I will apply to any polygon? Explain.

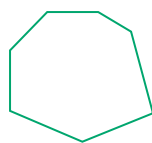
APPLY the Math

EXAMPLE 1

Reasoning about the sum of the interior angles of a polygon

convex polygon

A polygon in which each interior angle measures less than 180° .



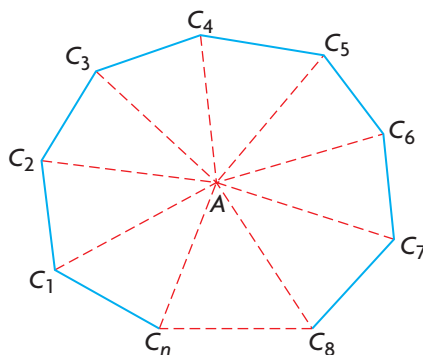
convex



non-convex
(concave)

Prove that the sum of the measures of the interior angles of any n -sided **convex polygon** can be expressed as $180^\circ(n - 2)$.

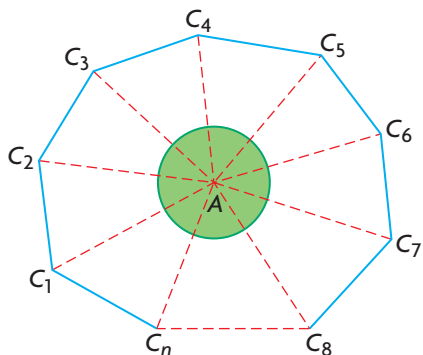
Viktor's Solution



I drew an n -sided polygon. I represented the n th side using a broken line. I selected a point in the interior of the polygon and then drew line segments from this point to each vertex of the polygon. The polygon is now separated into n triangles.

The sum of the measures of the angles in each triangle is 180° .

The sum of the measures of the angles in n triangles is $n(180^\circ)$.



Two angles in each triangle combine with angles in the adjacent triangles to form two interior angles of the polygon.

Each triangle also has an angle at vertex A . The sum of the measures of the angles at A is 360° because these angles make up a complete rotation. These angles do not contribute to the sum of the interior angles of the polygon.

The sum of the measures of the interior angles of the polygon, $S(n)$, where n is the number of sides of the polygon, can be expressed as:

$$S(n) = 180^\circ n - 360^\circ$$

$$S(n) = 180^\circ(n - 2)$$

The sum of the measures of the interior angles of a convex polygon can be expressed as $180^\circ(n - 2)$.

Your Turn

Explain why Viktor's solution cannot be used to show whether the expression $180^\circ(n - 2)$ applies to non-convex polygons.

EXAMPLE 2**Reasoning about angles in a regular polygon**

Outdoor furniture and structures like gazebos sometimes use a regular hexagon in their building plan. Determine the measure of each interior angle of a regular hexagon.

**Nazra's Solution**

Let $S(n)$ represent the sum of the measures of the interior angles of the polygon, where n is the number of sides of the polygon.

$$S(n) = 180^\circ(n - 2)$$

$$S(6) = 180^\circ[(6) - 2]$$

$$S(6) = 720^\circ$$

$$\frac{720^\circ}{6} = 120^\circ$$

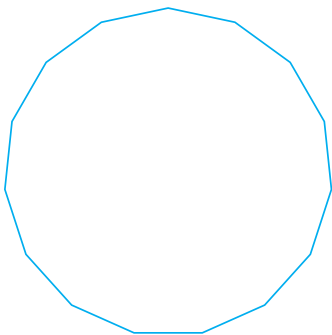
The measure of each interior angle of a regular hexagon is 120° .

A hexagon has six sides, so $n = 6$.

Since the measures of the angles in a regular hexagon are equal, each angle must measure $\frac{1}{6}$ of the sum of the angles.

Your Turn

Determine the measure of each interior angle of a regular 15-sided polygon (a pentadecagon).



EXAMPLE 3**Visualizing tessellations**

A floor tiler designs custom floors using tiles in the shape of regular polygons. Can the tiler use congruent regular octagons and congruent squares to tile a floor, if they have the same side length?

Vanessa's Solution

$$S(n) = 180^\circ(n - 2)$$

$$S(8) = 180^\circ[(8) - 2]$$

$$S(8) = 1080^\circ$$

$$\frac{1080^\circ}{8} = 135^\circ$$

The measure of each interior angle in a regular octagon is 135° .

The measure of each internal angle in a square is 90° .

Two octagons fit together, forming an angle that measures:

$$2(135^\circ) = 270^\circ.$$

This leaves a gap of 90° .

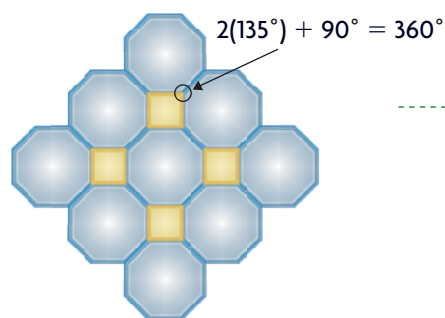
$$2(135^\circ) + 90^\circ = 360^\circ$$

A square can fit in this gap if the sides of the square are the same length as the sides of the octagon.

Since an octagon has eight sides, $n = 8$.

First, I determined the sum of the measures of the interior angles of an octagon. Then I determined the measure of each interior angle in a regular octagon.

I knew that three octagons would not fit together, as the sum of the angles would be greater than 360° .



I drew what I had visualized using dynamic geometry software.

The tiler can tile a floor using regular octagons and squares when the polygons have the same side length.

Your Turn

Can a tiling pattern be created using regular hexagons and equilateral triangles that have the same side length? Explain.

In Summary

Key Idea

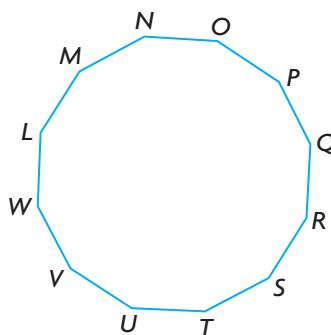
- You can prove properties of angles in polygons using other angle properties that have already been proved.

Need to Know

- The sum of the measures of the interior angles of a convex polygon with n sides can be expressed as $180^\circ(n - 2)$.
- The measure of each interior angle of a regular polygon is $\frac{180^\circ(n - 2)}{n}$.
- The sum of the measures of the exterior angles of any convex polygon is 360° .

CHECK Your Understanding

- a) Determine the sum of the measures of the interior angles of a regular dodecagon.
b) Determine the measure of each interior angle of a regular dodecagon.



- Determine the sum of the measures of the angles in a 20-sided convex polygon.
- The sum of the measures of the interior angles of an unknown polygon is 3060° . Determine the number of sides that the polygon has.

PRACTISING

- Honeybees make honeycombs to store their honey. The base of each honeycomb is roughly a regular hexagon. Explain why a regular hexagon can be used to tile a surface.



5. Is it possible to create a tiling pattern with parallelograms? Explain.
6. Determine the measure of each interior angle of a loonie.



7. Each interior angle of a regular convex polygon measures 140° .
 - a) Prove that the polygon has nine sides.
 - b) Verify that the sum of the measures of the exterior angles is 360° .
8.
 - a) Determine the measure of each exterior angle of a regular octagon.
 - b) Use your answer for part a) to determine the measure of each interior angle of a regular octagon.
 - c) Use your answer for part b) to determine the sum of the interior angles of a regular octagon.
 - d) Use the function

$$S(n) = 180^\circ(n - 2)$$
 to determine the sum of the interior angles of a regular octagon. Compare your answer with the sum you determined in part c).
9.
 - a) Wallace claims that the opposite sides in any regular hexagon are parallel. Do you agree or disagree? Justify your decision.
 - b) Make a conjecture about parallel sides in regular polygons.

Math in Action

"Circular" Homes

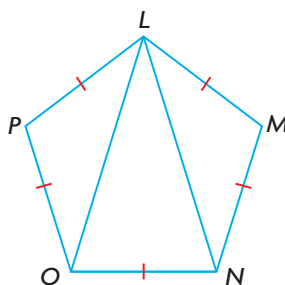
A building based on a circular floor plan has about 11% less outdoor wall surface area than one based on a square floor plan of the same area. This means less heat is lost through the walls in winter, lowering utility bills.

Most "circular" buildings actually use regular polygons for their floor plans.

- Determine the exterior angle measures of a floor plan that is a regular polygon with each of the following number of sides: 12, 18, 24. Explain why a building would be closer to circular as the number of sides increases.
- List some practical limitations on the number of sides a building could have.
- Based on the practical limitations, suggest an optimal number of sides for a home. Sketch a floor plan for a home with this number of sides.



10. $LMNOP$ is a regular pentagon.
- Determine the measure of $\angle OLN$.
 - What kind of triangle is $\triangle LON$?
Explain how you know.

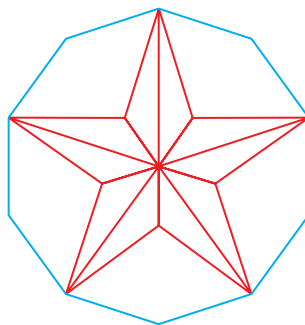


11. Sandy designed this logo for the jerseys worn by her softball team. She told the graphic artist that each interior angle of the regular decagon should measure 162° , based on this calculation:

$$S(10) = \frac{180^\circ(10 - 1)}{10}$$

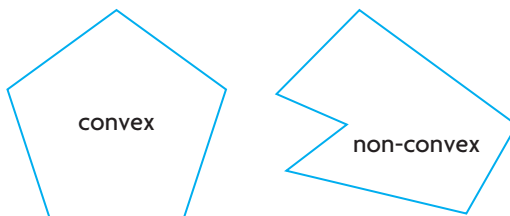
$$S(10) = \frac{1620^\circ}{10}$$

$$S(10) = 162^\circ$$

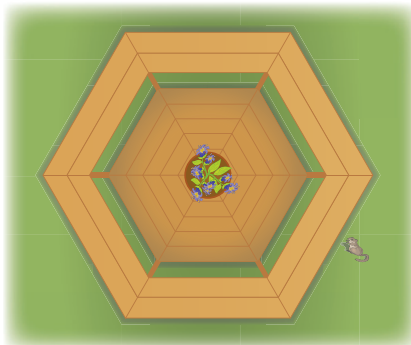


Identify the error she made and determine the correct angle.

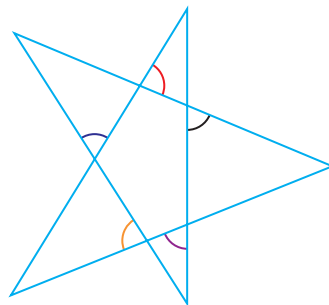
12. Astrid claims that drawing lines through a polygon can be used as a test to determine whether the polygon is convex or non-convex (concave).



- Describe a test that involves drawing a single line.
 - Describe a test that involves drawing diagonals.
13. Martin is planning to build a hexagonal picnic table, as shown.
- Determine the angles at the ends of each piece of wood that Martin needs to cut for the seats.
 - How would these angles change if Martin decided to make an octagonal table instead?

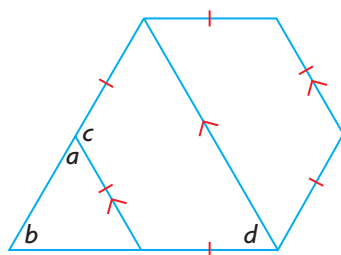


14. Three exterior angles of a convex pentagon measure 70° , 60° , and 90° . The other two exterior angles are congruent. Determine the measures of the interior angles of the pentagon.
15. Determine the sum of the measures of the indicated angles.

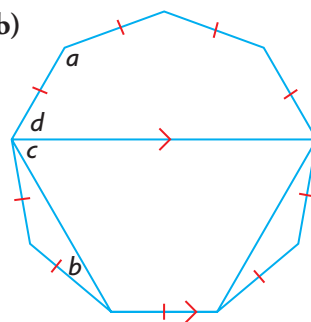


16. In each figure, the congruent sides form a regular polygon. Determine the values of a , b , c , and d .

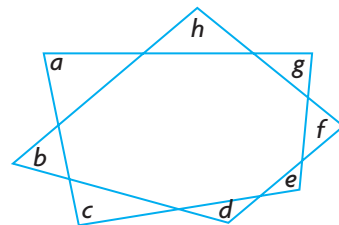
a)



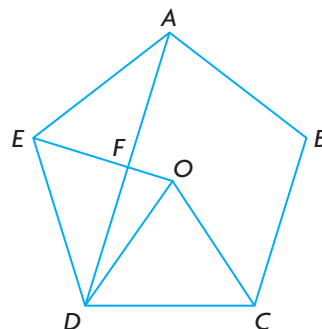
b)



17. Determine the sum of the measures of the indicated angles.



18. Given: $ABCDE$ is a regular pentagon with centre O .
 $\triangle EOD$ is isosceles, with $EO = DO$.
 $DO = CO$
 Prove: $\triangle EFD$ is a right triangle.



Closing

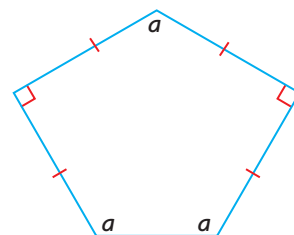
19. The function representing the sum of the measures of the interior angles of a polygon with n sides is:

$$S(n) = 180^\circ(n - 2)$$

Explain how the expression on the right can be deduced by considering a polygon with n sides.

Extending

20. A pentagon tile has two 90° angles. The other three angles are equal. Is it possible to create a tiling pattern using only this tile? Justify your answer.
21. Each interior angle of a regular polygon is five times as large as its corresponding exterior angle. What is the common name of this polygon?



History | Connection

Buckyballs—Polygons in 3-D

Richard Buckminster “Bucky” Fuller (1895–1983) was an American architect and inventor who spent time working in Canada. He developed the geodesic dome and built a famous example, now called the Montréal Biosphere, for Expo 1967. A spin-off from Fuller’s dome design was the buckyball, which became the official design for the soccer ball used in the 1970 World Cup.

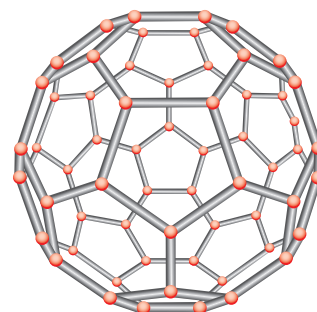
In 1985, scientists discovered carbon molecules that resembled Fuller’s geodesic sphere. These molecules were named fullerenes, after Fuller.



The Montréal Biosphere and its architect



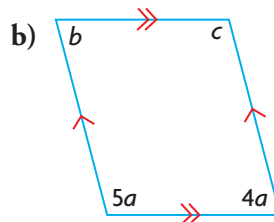
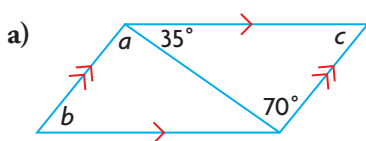
FIFA soccer ball, 1970



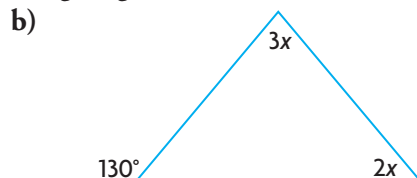
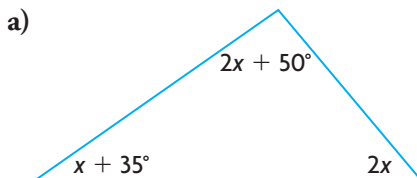
Carbon molecule, C_{60}

- A. Identify the polygons that were used to create the buckyball.
- B. Predict the sum of the three interior angles at each vertex of the buckyball. Check your prediction.
- C. Explain why the value you found in part B makes sense.

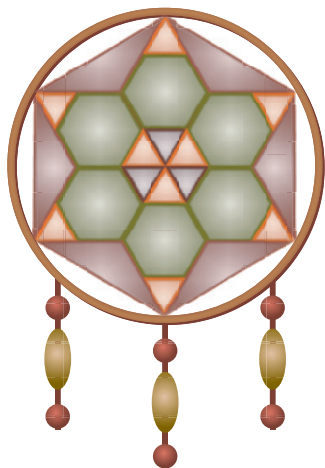
1. Determine the values of a , b , and c .



2. Determine the value of x in the following diagrams.

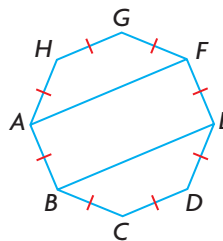


3. a) Construct a pair of parallel lines and a transversal using a protractor and a straight edge.
 b) Label your sketch, and then show by measuring that the alternate interior angles in your sketch are equal.
 c) Identify all the pairs of equal angles in your sketch.
4. Joyce is an artist who uses stained glass to create sun catchers, which are hung in windows. Joyce designed this sun catcher using triangles and regular hexagons. Determine the measure of the interior angles of each different polygon in her design.

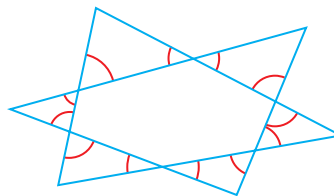


5. $ABCDEFGH$ is a regular octagon.

- a) Draw an exterior angle at vertex C .
 b) Determine the measure of the exterior angle you drew.
 c) Prove: $AF \parallel BE$



6. Determine the sum of the indicated angles.



WHAT DO You Think Now? Revisit **What Do You Think?** on page 69. How have your answers and explanations changed?

FREQUENTLY ASKED Questions

Q: How are angle properties in convex polygons developed using other angle properties?

A1: If you draw a line through one of the vertices of a triangle parallel to one of the sides, you will create two transversals between two parallel lines. You can use the angle property that alternate interior angles are equal to show that the sum of the measures of the three interior angles of a triangle is 180° .

Study Aid

- See Lesson 2.3.
- Try Chapter Review Questions 7 and 8.

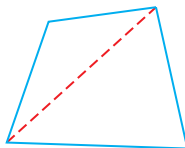
A2: The sum of the measures of the angles in any triangle is 180° . You can use this property to develop a relationship between the number of sides in a convex polygon and the sum of the measures of the interior angles of the polygon.

Study Aid

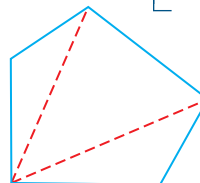
- See Lesson 2.4, Example 1.
- Try Chapter Review Questions 9 and 10.



$n = 3$, triangle
Sum of Interior Angles = $(180^\circ) 1$



$n = 4$, quadrilateral
Sum of Interior Angles = $(180^\circ) 2$



$n = 5$, pentagon
Sum of Interior Angles = $(180^\circ) 3$

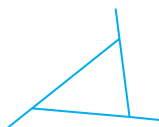
Using inductive reasoning, you can show that for any polygon with n sides, the sum of the measures of the interior angles, $S(n)$, can be determined using the relationship:

$$S(n) = 180^\circ(n - 2)$$

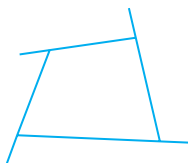
A3: When two angles share a vertex on a straight line, the angles are supplementary. You can use this angle property, along with the angle measure sum property for convex polygons, to develop a property about the exterior angles of a convex polygon. If you extend each side of a convex polygon, you will create a series of exterior angles.

Study Aid

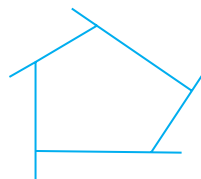
- See Lesson 2.4.
- Try Chapter Review Question 10.



$n = 3$, triangle
Sum of Interior Angles = $(180^\circ) 1$
Sum of Exterior Angles:
 $3(180^\circ) - (180^\circ) 1 = 360^\circ$



$n = 4$, quadrilateral
Sum of Interior Angles = $(180^\circ) 2$
Sum of Exterior Angles:
 $4(180^\circ) - 2(180^\circ) = 360^\circ$



$n = 5$, pentagon
Sum of Interior Angles = $(180^\circ) 3$
Sum of Exterior Angles:
 $5(180^\circ) - 3(180^\circ) = 360^\circ$

Using inductive reasoning, you can show that for any polygon with n sides, the sum of the exterior angles, $A(n)$, is determined using the relationship:

$$A(n) = n(180^\circ) - (n - 2)180^\circ$$

$$A(n) = 360^\circ$$

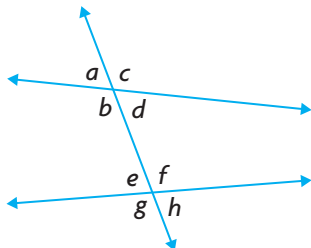
PRACTISING

Lesson 2.1

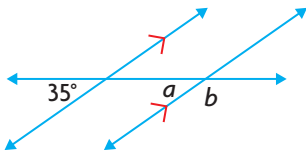
1. Kamotiqs are sleds that are dragged behind vehicles, such as snowmobiles, over snow and sea ice. Identify a set of parallel lines and a transversal in the photograph of a kamotiq.



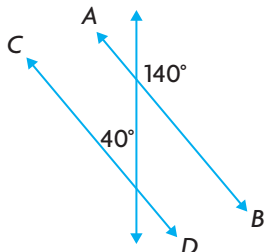
2. a) Name the pairs of corresponding angles.



- b) Are any of the pairs you identified in part a) equal? Explain.
 - c) How many pairs of supplementary angles can you see in the diagram? Name one pair.
 - d) Are there any other pairs of equal angles? If so, name them.
3. Determine the values of a and b .

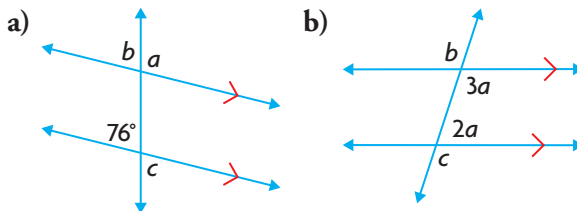


4. Is AB parallel to CD ? Explain how you know.



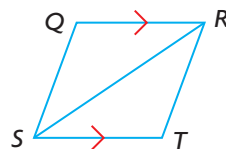
Lesson 2.2

5. Determine the values of a , b , and c .



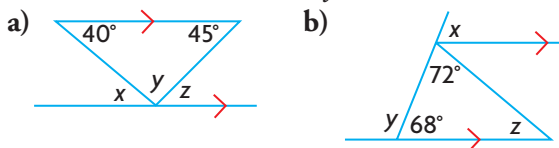
6. a) Construct a pair of parallel lines using a straight edge and a compass.
b) Explain two different ways you could verify that your lines are parallel using a protractor.

7. Given: $QR \parallel ST$
 $\angle QRS = \angle TRS$
Prove: $ST = TR$

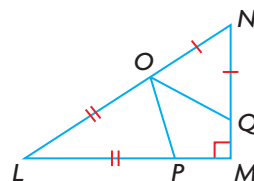


Lesson 2.3

8. Determine the values of x , y , and z .

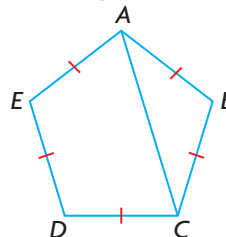


9. Given: $LM \perp MN$
 $LP = LO$
 $NO = NQ$
Prove: $\angle POQ = 45^\circ$



Lesson 2.4

10. a) Determine the sum of the measures of the interior angles of a 15-sided regular polygon.
b) Show that each exterior angle measures 24° .
11. Given: $ABCDE$ is a regular pentagon.
Prove: $AC \parallel ED$



Designing Logos

A logo is often used to represent an organization and its products or services. The combination of shapes, colours, and fonts that are used in a logo is unique, making the logo stand out from the logos of other organizations. Logos transmit a message about the nature of an organization and its special qualities.

Logos appeal to our most powerful sense—the visual. After seeing a logo a few times, consumers can often identify the organization instantaneously and without confusion when they see the logo again.



? How can you design a logo that incorporates parallel lines and polygons?

- A. What kind of organization do you want to design a logo for? What message do you want to convey with your logo?
- B. How can you use shapes and lines to convey the message?
- C. Draw a sketch of your logo. Mark parallel lines, equal sides, and angle measures. Explain how you know that each of your markings is correct.
- D. Use your sketch to draw the actual logo you would present to the board of directors of the organization.

Task	Checklist
	✓ Did you clearly explain your message?
	✓ Did you provide a labelled sketch and a finished design?
	✓ Did you provide appropriate reasoning?

Selecting Your Research Topic

To decide what to research, you can start by thinking about subjects and then consider specific topics. Some examples of subjects and topics are shown in the table below.

Subject	Topic
entertainment	<ul style="list-style-type: none"> • effects of new devices • file sharing
health care	<ul style="list-style-type: none"> • doctor and/or nurse shortages • funding
post-secondary education	<ul style="list-style-type: none"> • entry requirements • graduate success
history of the West and North	<ul style="list-style-type: none"> • relations between First Nations • immigration

It is important to take the time to consider several topics carefully before selecting a topic for your research project. Below is a list of criteria that will help you to determine if a topic you are considering is suitable.

Criteria for Selecting Your Research Topic

- **Does the topic interest you?**

You will be more successful if you choose a topic that interests you.

You will be more motivated to do the research, and you will be more attentive while doing the research. As well, you will care more about the conclusions you draw.

- **Is the topic practical to research?**

If you decide to use first-hand data, can you generate the data in the time available, with the resources you have? If you decide to use second-hand data, are there multiple sources of data? Are these sources reliable, and can you access them in a timely manner?

- **Is there an important issue related to the topic?**

Think about the issues related to the topic. If there are many viewpoints on an issue, you may be able to gather data that support some viewpoints but not others. The data you collect should enable you to come to a reasoned conclusion.

- **Will your audience appreciate your presentation?**

Your topic should be interesting to others in your class, so they will be attentive during your presentation. Avoid a topic that may offend anyone in your class.

PROJECT EXAMPLE

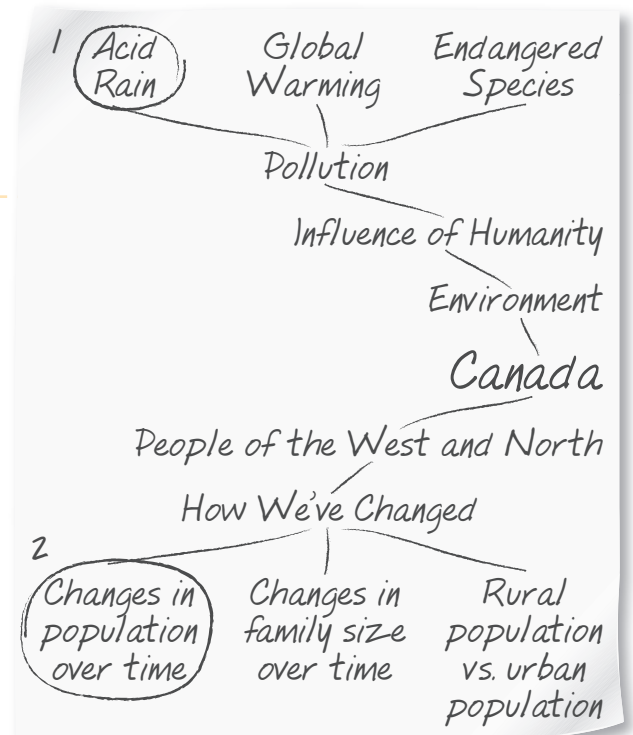
Choosing a topic

Sarah identified some subjects that interest her. Below she describes how she went from subjects to topics and then to one research topic.

Sarah's Research Topic

I identified five subjects that interested me and seemed to be worth exploring: health care, entertainment, post-secondary education, peoples of the West and North, and the environment. I used a mind map to organize my thoughts about each subject into topics. (Part of my mind map is shown here.) This gave me a list of possible topics to choose from.

I needed to narrow down my list by evaluating each topic. I wanted a topic that would be fun to research and might interest others in my class. I also wanted a topic that had lots of available data. Finally, I wanted a topic that involved an issue for a chance to come to a useful conclusion. I circled the two topics that seemed most likely to work. Then I did an initial search for information about these topics.



1. I spent some time researching the history of acid rain in Canada. I discovered that the problem was identified in the 1960s, but it has been a problem mostly in Eastern Canada. However, I did come across some newspaper articles and journal reports warning that acid rain may become a problem in the West. As I searched for data to support this claim, I found little historical data. I concluded that this topic would be too difficult to research.
2. I knew that the federal government conducts a census every four years to collect information about the people who live in Canada, so I was confident that I would be able to find lots of historical data. When I searched the Internet, I found several data sources for Canada's population. I feel that changes in the population of the West and North would make a good topic for my project.

Your Turn

- A. Choose several subjects that interest you. Then make a list of topics that are related to each subject. A graphic organizer, such as a concept web or mind map, is useful for organizing your thoughts.
- B. Once you have chosen several topics, do some research to see which topic would best support a project. Of these, choose the one that you think is the best. Refer to "Criteria for Selecting Your Research Topic."

1. a) What is a conjecture?
b) Explain how inductive reasoning can be used to make a conjecture.
c) How can inductive reasoning lead to a false conjecture? Explain using an example.
2. Maja gathered the following evidence and noticed a pattern.

$$1^3 + 2^3 + 3^3 = 36 \qquad 7^3 + 8^3 + 9^3 = 1584$$

$$3^3 + 4^3 + 5^3 = 216 \qquad 10^3 + 11^3 + 12^3 = 4059$$

She made a conjecture: The sum of the cubes of any three consecutive positive integers is a multiple of 9. Is her conjecture reasonable? Develop evidence to test her conjecture.
3. How many counterexamples are needed to disprove a conjecture? Explain using an example.
4. Sidney claims that the difference between two positive integers is always a positive integer. Do you agree or disagree? Justify your decision.
5. a) Use inductive reasoning to make a conjecture about the sum of two odd numbers.
b) Use deductive reasoning to prove your conjecture.
6. Sung Lee says that this number trick always ends with the number 8: Choose a number. Double it. Add 9. Add the number you started with. Divide by 3. Add 5. Subtract the number you started with. Prove that Sung Lee is correct.
7. a) Determine the rule for the number of circles in the n th figure. Use your rule to determine the number of circles in the 15th figure.

Figure 1



Figure 2



Figure 3



Figure 4

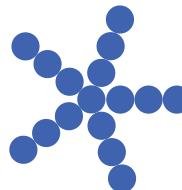
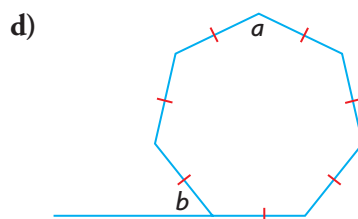
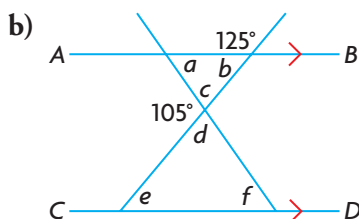
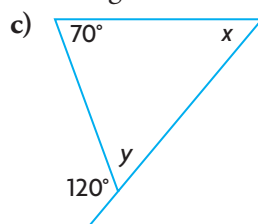
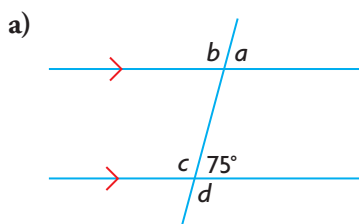


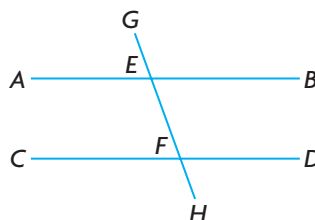
Figure Number	1	2	3	4	...	n	...	15
Number of Circles	1	6	11	16	

- b) Did you use inductive or deductive reasoning to answer part a)? Explain.

8. Prove the following conjecture for all three-digit numbers: If the digit in the ones place of a three-digit number is 0, the number is divisible by 10.
9. There are three switches in a hallway, all in the off position. Each switch corresponds to one of three light bulbs in a room with a closed door. You can turn the switches on and off, and you can leave them in any position. You can enter the room only once. Describe how you would identify which switch corresponds to which light bulb.
10. Determine the measure of each indicated angle.



11. What information would you need to prove that AB is parallel to CD ?



12. This photograph is an aerial view of the Pentagon in Washington, D.C.
- Determine the sum of the interior angles of the courtyard.
 - Determine the measure of each interior angle of the courtyard.
 - Determine the sum of the exterior angles of the building.
13. The sum of the measures of the angles in any triangle is 180° . Explain how this property can be used to develop the formula for the sum of the interior angles of any convex polygon, $S(n) = 180^\circ(n - 2)$.

